# Representation Before Geometry: A Categorical-Condensed Foundation for Geometry, Quantum Theory, and Statistical Physics

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**Abstract**

This paper asks how much of familiar physics follows after category, representation, realization, and coarse-graining data have been separated instead of folded into the word "emergence." The answer is deliberately limited. A categorical context signature can be stated before spacetime. Condensed mathematics can supply an optional ambient realization for some topological and limit constructions. Representation data can organize symmetry, observables, and typed outcome labels. None of those declarations by itself produces a Lorentzian manifold, operational probabilities, complex Hilbert space, or a hard-sphere kinetic limit.

The six papers in this series are composed through a machine-checked, content-addressed dependency graph. Its authoring snapshot contains 42 registered claims, 78 assumptions, 49 counterexamples or obstructions, 78 source records, 13 typed interfaces, 42 evidence records, and exact terminal review bindings. The graph checks closed-world registry coverage, artifact hashes, review freshness, claim-status compatibility, interface fingerprints, semantic terms, hard cycles, branch satisfiability, and target closure digests. A prospective Lorentzian-to-measurement edge is excluded from the hard closure because making it hard would create a circular reconstruction. The condensed interface is optional and requires a receipt for every use. There is no hard Paper 5 to Paper 6 edge because no conversion from an operational or Hilbert-space state to the classical grand-canonical hard-sphere state has been supplied.

The resulting synthesis has three branch statements, not one aggregate theorem. The geometry branch is a conditional realization schema whose project bridge claims remain **OPEN**. The quantum branch is also **OPEN**at its reconstruction bridges and keeps outcome labels, operational effects, probabilities, instruments, and interpretations separate. The kinetic branch contains narrow **PROVED**and **COMPUTATIONAL**finite

results together with precisely scoped **CONDITIONAL**imported consequences. No certificate shows that one model satisfies all three branch assumption packages. The strongest defensible thesis is therefore conditional and branch local: established physical laws can occur as realizations when their exact hypotheses and conversion maps are supplied. The repository does not prove that those laws are forced by categorical locality alone.

# Contents

# 1 The composition problem

The phrase "representation before geometry" can be read in two ways. One reading is methodological: begin with typed systems, processes, gluing, and representations, then state separately what must be added to obtain a geometric or probabilistic model. The second reading is a strong derivation claim: spacetime, quantum mechanics, and statistical mechanics are forced by the pre-geometric axioms. This series supports the first reading. It does not establish the second.

That distinction changes the logical shape of a synthesis. A list of six accepted manuscripts is not a proof that their headline claims compose. Several manuscripts correctly conclude that central bridge claims remain **OPEN**. A compiled Lean structure may store an implication supplied by its caller without proving the implication. A passing Haskell suite may check finite values without deciding a continuum theorem. A published reconstruction theorem may start with precisely the geometry or probability that the project hopes to recover. All of these artifacts are useful, but they have different types.

The unit of composition in this paper is a versioned claim instance $c@v$. Let $T@w$ be a branch target. The graph accepts $c@v$ into $T@w$ only when

$$\begin{aligned}\mathrm{Admissible}(c@v, T@w) \quad \Longleftrightarrow \quad & \mathrm{ExactStatement}(c@v) \\ & \wedge \mathrm{StatusFitsEvidence}(c@v) \\ & \wedge \mathrm{HardInputsResolved}(c@v) \\ & \wedge \mathrm{TypesCompatible}(c@v, T@w) \\ & \wedge \mathrm{AssumptionsPreserved}(c@v, T@w) \\ & \wedge \mathrm{ScopeExplicit}(c@v, T@w) \\ & \wedge \mathrm{ReviewCurrent}(c@v) \\ & \wedge \mathrm{InClosureDigest}(c@v, T@w).\end{aligned} \tag{1}$$

Each predicate is represented by data that the validator can inspect. No predicate is inferred from a paper title.

*Warning* 1.1 (The non-theorem). There is no theorem in this repository asserting the existence of one model that simultaneously realizes the geometry, quantum, and kinetic branches. The authoring target has scope `SEPARATE_REALIZATIONS`. A `SAME_REALIZATION` target would require a joint-model evidence record, which is absent.

The warning governs the quantifiers. The geometry branch uses a smooth Lorentzian realization with causal, projective, clock, differentiability, and variational data. The quantum branch uses a finite operational model or a finite homogeneous self-dual comparison route, then adds positivity and probability hypotheses. The kinetic branch uses classical deterministic hard spheres, a grand-canonical law, Boltzmann-Grad scaling, and a regular Boltzmann solution. These are separate witness types. Similar notation for "state" does not identify them.

# 2 What the first three papers export

## 2.1 Categorical locality

Paper 1 starts with selected contexts, processes, composition, covers, and descent data. Its category-theoretic comparators include the cobordism viewpoint of Atiyah [3], higher-categorical locality, and standard descent language [32, 42]. The safe graph exports are signatures:

- `P1-I-CTX-SIGNATURE`, a parameterized context and process signature;
- `P1-I-COVER-SIGNATURE`, a cover-family signature; and
- `P1-I-DESCENT-SIGNATURE`, descent data together with a type for a supplied effectiveness witness.

Their availability is `DECLARED`. A declaration shows that later papers can state their inputs without type confusion. It does not choose a physical category, a coverage, or a causal relation.

Paper 1 also contains narrower paper-local mathematics about composition, finite symmetric reductions, and supplied effective-descent witnesses. Those statements do not prove the stable physical claims `P1-C01` through `P1-C04`. All four remain **OPEN**. In particular, a coverage is not a causal order, and descent does not select dimension, topology, or metric.

## 2.2 The condensed option

Paper 2 studies condensed and solid objects as an ambient setting for topological, distributional, and limit constructions [11, 37]. Its proposed export, `P2-I-CONDENSED-REALIZATION`, is marked **OPEN**. The graph treats it as optional.

Every optional use needs a receipt. The receipt must name the source category, target category, functor version, preserved operation, size convention, and exact consumer. The present graph has three prospective receipt sites, one in each realization branch. None is a hard dependency. This prevents two opposite mistakes. Later papers do not acquire condensed structure merely by citing Paper 2, and they are not forced to use condensed mathematics when ordinary finite or smooth structures suffice.

Condensation is not geometry selection. A condensed avatar of an object does not gain a Lorentzian metric, a probability interpretation, or a dynamics. That boundary is recorded by `CE-P2-01`. Full faithfulness also depends on the selected source category, as recorded by `CE-P2-02`.

### 2.3 Representation and pre-geometric measurement

Paper 3 separates several outputs that are often compressed into one sentence. Tannakian and Doplicher-Roberts reconstruction require rigidity, symmetry, conjugates or duals, a suitable unit, and a fibre functor [15, 18, 41]. A symmetry object does not select an observable algebra. An action on observables does not select dynamics. An invariant observable is not automatically conserved under an unrelated flow.

The graph exports two open interfaces:

`P3-I-OBSERVABLE` : context-indexed observables and a declared symmetry action,

`P3-I-OUTCOME` : typed outcome labels with monoidal composition.

The second line contains no probability map. This agrees with the categorical treatment of outcome objects in Abramsky and Heunen [1], while avoiding an identification with effects or Born probabilities.

The invariant-state claim `P3-C03` is **OBSTRUCTED**. A nonamenable translation action supplies a concrete obstruction to the claim that every continuous automorphic action has an invariant state [13, 14, 43]. The obstruction does not deny invariant states for compact or amenable cases. It requires the missing hypothesis to be visible.

## 3 The dependency graph

### 3.1 Node and edge types

The canonical file is `research/dependency-graph.v1.json`. Its schema version is `foundational-reformulation/dependency-graph/v1`. The graph contains the following node classes:

1. papers and file artifacts, each artifact carrying a SHA-256 digest;
2. registry claims, assumptions, obstructions, source keys, and terms;
3. typed interfaces with revision, branch, availability, type fingerprint, variance, composition law, size convention, and semantic term bindings;
4. evidence records with a class, exact artifact, locator, supported claim revision, source keys, and assumptions;
5. terminal review records with review scope, reviewed input hashes, closure digest, verdict, and review-file hash; and

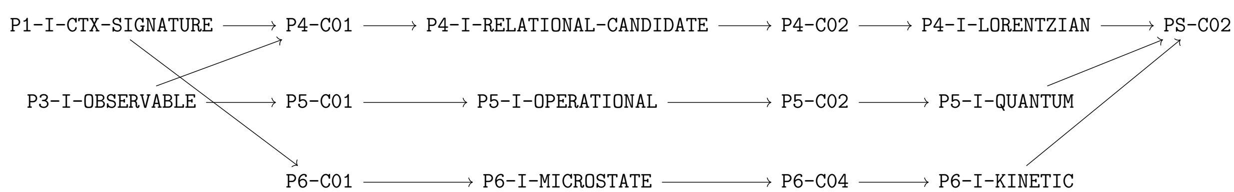
6. composition targets with realization scope, root assumptions, allowed statuses, transitive node list, and closure digest.

The 35 terminology rows are represented exactly. Three additional term nodes refine the single registry word "state" without changing its registered meaning. They distinguish an algebraic positive functional, an operational state, and a kinetic many-body law or density. A consumer cannot silently use one refinement in place of another.

Hard edges express premises that enter a target closure. A hard `CONSUMES` edge binds an exact interface revision to a claim. If fingerprints or semantic terms differ, the validator requires conversion evidence. Prospective edges document research directions but do not enter the hard acyclic closure.

## 3.2 The branch graph

The hard compositional spine is




Paper 3's outcome interface also enters `P5-C01`. The diagram omits secondary edges to keep it readable, but the JSON closure includes them. Every consumed interface has a matching hard edge.

Two arrows are intentionally absent. First, the Lorentzian interface has only a prospective edge back to `P3-C05`. Making that edge hard would ask Paper 3 to use a geometry that Paper 4 itself tries to realize from Papers 1 through 3. Second, there is no Paper 5 to Paper 6 hard edge. A density operator, operational state, or Hilbert vector has no registered conversion to the grand-canonical classical hard-sphere law used in Paper 6.

## 3.3 The validator

The command

```
node scripts/validate-dependency-graph.mjs
  -graph research/dependency-graph.v1.json
    -target TARGET-SYNTHESIS-AUTHORING
```

checks the graph against the live registries and artifacts. Its adversarial suite covers duplicate IDs, stale artifacts, stale reviews, hard cycles, missing same-realization evidence, incompatible interface fingerprints, semantic-term drift, status-evidence mismatch, missing hard consumption edges, closure tampering, and branch-specific assumption incompatibility. Prospective edges are deliberately ignored by the hard-cycle check.

The validator checks a finite JSON object against the repository's composition contract. Physical existence is outside that contract, so a passing run supplies no model for an **OPEN**assumption. This is why `PS-C01` and `PS-C03` remain **OPEN**even though the current authoring target passes the mechanical graph checks.

# 4 Complete claim-status matrix

The table below is a literal projection of the claim register. The validator requires every registered claim to occur exactly once with its exact status. No manuscript review, compilation result, or graph pass changes these words.

| Claim revision | Registered status |
|---|---|
| P1-C01 | OPEN |
| P1-C02 | OPEN |
| P1-C03 | OPEN |
| P1-C04 | OPEN |
| P2-C01 | OPEN |
| P2-C02 | OPEN |
| P2-C03 | OPEN |
| P2-C04 | OPEN |
| P3-C01 | OPEN |
| P3-C02 | OPEN |
| P3-C03 | OBSTRUCTED |
| P3-C04 | OPEN |
| P3-C05 | OPEN |
| P4-C01 | OPEN |
| P4-C02 | OPEN |
| P4-C03 | OPEN |
| P4-C04 | OPEN |
| P4-C05 | OBSTRUCTED |
| P5-C01 | OPEN |
| P5-C02 | OPEN |
| P5-C03 | OPEN |
| P5-C04 | OPEN |
| P5-C05 | OBSTRUCTED |
| P5-C06 | OPEN |
| P6-C01 | OPEN |
| P6-C01a | PROVED |
| P6-C01b | PROVED |
| P6-C01c | OPEN |
| P6-C02 | OPEN |
| P6-C02a | PROVED |
| P6-C02b | COMPUTATIONAL |

| Claim revision | Registered status |
|---|---|
| P6-C03 | CONDITIONAL |
| P6-C04 | CONDITIONAL |
| P6-C05 | OPEN |
| P6-C05a | PROVED |
| P6-C05b | CONDITIONAL |
| P6-C05c | CONDITIONAL |
| P6-C06 | CONDITIONAL |
| PS-C01 | OPEN |
| PS-C02 | OPEN |
| PS-C03 | OPEN |
| PS-C04 | OPEN |

Table 1: All 42 registered claims. The status vocabulary is closed.

There are 29 **OPEN**claims, four **PROVED**claims, five **CONDITIONAL**claims, one **COMPUTATIONAL**claim, and three **OBSTRUCTED**claims. The proved and computational entries are narrow finite statements in Paper 6. The conditional entries preserve the complete hypothesis package of imported kinetic results. None upgrades a synthesis claim.

## 5 Complete assumption map

The authoring target roots all 78 assumptions. Rooting them is conservative: the closure cannot hide an unvalidated premise merely because no current positive claim consumes it. The only established assumption is P1-A06. The remaining 77 are **UNVALIDATED**.

| ID | Paper or branch | Needed by | State |
|---|---|---|---|
| P1-A01 | categorical-locality | P1-C01 | UNVALIDATED |
| P1-A02 | categorical-locality | P1-C01, P1-C02 | UNVALIDATED |
| P1-A03 | categorical-locality | P1-C02 | UNVALIDATED |
| P1-A04 | categorical-locality | P1-C02 | UNVALIDATED |
| P1-A05 | categorical-locality | P1-C03 | UNVALIDATED |
| P1-A06 | categorical-locality | P1-C03 | ESTABLISHED |
| P1-A07 | categorical-locality | P1-C04 | UNVALIDATED |
| P2-A01 | condensed-contexts | P2-C01 | UNVALIDATED |
| P2-A02 | condensed-contexts | P2-C01, P2-C02 | UNVALIDATED |
| P2-A03 | condensed-contexts | P2-C01 | UNVALIDATED |
| P2-A04 | condensed-contexts | P2-C02 | UNVALIDATED |

| ID | Paper or branch | Needed by | State |
|---|---|---|---|
| P2-A05 | condensed-contexts | P2-C02 | UNVALIDATED |
| P2-A06 | condensed-contexts | P2-C03 | UNVALIDATED |
| P2-A07 | condensed-contexts | P2-C03 | UNVALIDATED |
| P2-A08 | condensed-contexts | P2-C04 | UNVALIDATED |
| P3-A01 | representation | P3-C01 | UNVALIDATED |
| P3-A02 | representation | P3-C01 | UNVALIDATED |
| P3-A03 | representation | P3-C01 | UNVALIDATED |
| P3-A04 | representation | P3-C02 | UNVALIDATED |
| P3-A05 | representation | P3-C02 | UNVALIDATED |
| P3-A06 | representation | P3-C03 | UNVALIDATED |
| P3-A07 | representation | P3-C03 | UNVALIDATED |
| P3-A08 | representation | P3-C04 | UNVALIDATED |
| P3-A09 | representation | P3-C05 | UNVALIDATED |
| P4-A01 | geometry | P4-C01 | UNVALIDATED |
| P4-A02 | geometry | P4-C01 | UNVALIDATED |
| P4-A03 | geometry | P4-C01, P4-C02 | UNVALIDATED |
| P4-A04 | geometry | P4-C02 | UNVALIDATED |
| P4-A05 | geometry | P4-C02, P4-C03 | UNVALIDATED |
| P4-A06 | geometry | P4-C02 | UNVALIDATED |
| P4-A07 | geometry | P4-C03 | UNVALIDATED |
| P4-A08 | geometry | P4-C04 | UNVALIDATED |
| P4-A09 | geometry | P4-C04 | UNVALIDATED |
| P4-A10 | geometry | P4-C04 | UNVALIDATED |
| P4-A11 | geometry | P4-C04 | UNVALIDATED |
| P4-A12 | geometry | P4-C05 | UNVALIDATED |
| P5-A01 | quantum | P5-C01, P5-C02 | UNVALIDATED |
| P5-A02 | quantum | P5-C01, P5-C02 | UNVALIDATED |
| P5-A03 | quantum | P5-C01, P5-C02 | UNVALIDATED |
| P5-A04 | quantum | P5-C02 | UNVALIDATED |
| P5-A05 | quantum | P5-C02 | UNVALIDATED |
| P5-A06 | quantum | P5-C02 | UNVALIDATED |
| P5-A07 | quantum | P5-C02 | UNVALIDATED |
| P5-A08 | quantum | P5-C02 | UNVALIDATED |
| P5-A09 | quantum | P5-C02 | UNVALIDATED |
| P5-A10 | quantum | P5-C04 | UNVALIDATED |
| P5-A11 | quantum | P5-C04 | UNVALIDATED |
| P5-A12 | quantum | P5-C05 | UNVALIDATED |
| P5-A13 | quantum | P5-C06 | UNVALIDATED |

| ID | Paper or branch | Needed by | State |
|---|---|---|---|
| P5-A14 | quantum | P5-C03 | UNVALIDATED |
| P5-A15 | quantum | P5-C03 | UNVALIDATED |
| P5-A16 | quantum | P5-C03 | UNVALIDATED |
| P5-A17 | quantum | P5-C03 | UNVALIDATED |
| P5-A18 | quantum | P5-C03 | UNVALIDATED |
| P5-A19 | quantum | P5-C03 | UNVALIDATED |
| P5-A20 | quantum | none | UNVALIDATED |
| P5-A21 | quantum | none | UNVALIDATED |
| P5-A22 | quantum | none | UNVALIDATED |
| P5-A23 | quantum | none | UNVALIDATED |
| P6-A01 | kinetic | P6-C01, P6-C01c, P6-C03, P6-C04 | UNVALIDATED |
| P6-A02 | kinetic | P6-C01, P6-C01c | UNVALIDATED |
| P6-A03 | kinetic | P6-C01, P6-C01c | UNVALIDATED |
| P6-A04 | kinetic | P6-C02 | UNVALIDATED |
| P6-A05 | kinetic | P6-C02, P6-C02b | UNVALIDATED |
| P6-A06 | kinetic | P6-C03, P6-C04 | UNVALIDATED |
| P6-A07 | kinetic | P6-C03, P6-C04 | UNVALIDATED |
| P6-A08 | kinetic | P6-C03, P6-C04 | UNVALIDATED |
| P6-A09 | kinetic | P6-C03, P6-C04 | UNVALIDATED |
| P6-A10 | kinetic | P6-C05, P6-C05b, P6-C05c | UNVALIDATED |
| P6-A11 | kinetic | P6-C05, P6-C05b, P6-C05c | UNVALIDATED |
| P6-A12 | kinetic | P6-C03, P6-C04, P6-C06 | UNVALIDATED |
| PS-A01 | synthesis | PS-C01 | UNVALIDATED |
| PS-A02 | synthesis | PS-C01 | UNVALIDATED |
| PS-A03 | synthesis | PS-C02 | UNVALIDATED |
| PS-A04 | synthesis | PS-C02 | UNVALIDATED |
| PS-A05 | synthesis | PS-C03 | UNVALIDATED |
| PS-A06 | synthesis | PS-C03 | UNVALIDATED |
| PS-A07 | synthesis | PS-C04 | UNVALIDATED |

Table 2: The complete assumption-to-claim map.

The four Paper 5 assumptions with no current positive claim are retained because they guard excluded dynamics and measurement inferences. An unconsumed assumption is not permission to smuggle its conclusion into prose. It records a boundary that may become relevant when a successor claim is stated.

# 6 Measurement before and after realization

## 6.1 Pre-geometric measurement data

Before a geometry or operational theory has been chosen, an outcome object is a typed label with declared composition. For a context $X$, write

$$\mathsf{Outcome}(X)$$

for its outcome-label type. A monoidal comparison may provide

$$\mathsf{Outcome}(X \otimes Y) \longrightarrow \mathsf{Outcome}(X) \otimes \mathsf{Outcome}(Y),$$

with the direction and coherence fixed by the chosen convention. This construction can describe how labels combine. It cannot assign a number to an outcome.

An observable algebra is a different interface. A state on such an algebra, when it exists, has algebraic positivity and normalization conditions. An effect is an ordered element. A pairing

$$\omega : \mathsf{Effect}(X) \longrightarrow [0, 1]$$

becomes an operational probability only after an interpretation identifies effects with tests and identifies $\omega(e)$ with observed frequencies or betting weights. Paper 3 does not supply that interpretation. The distinctions are enforced by `CE-P3-09` and `CE-P5-05`.

## 6.2 Ordinary measurement

Ordinary measurement requires a realized system and a test procedure. The quantum branch uses an explicit strong-monoidal bridge from the Paper 3 observable and outcome interfaces into an operational-probabilistic theory. That bridge is the content of `P5-C01`, which remains **OPEN**. After the bridge, one can ask whether the operational axioms of Chiribella et al. [10] reconstruct finite complex quantum theory. Before the bridge, that question is ill typed.

The geometry branch supports a different notion of ordinary measurement: a test localized in a realized spacetime region. Connecting that notion back to `P3-I-OUTCOME` requires `P3-C05`. The only graph edge from the Lorentzian interface to that claim is prospective. This preserves the direction of explanation:

$$\text{pre-geometric label} \overset{\text{realization map}}{\dashrightarrow} \text{localized test} \overset{\text{evaluation}}{\dashrightarrow} .$$

Both dashed arrows are obligations. Neither is notation.

## 6.3 What a POVM does not settle

Even after a Hilbert-space realization, a positive operator-valued measure fixes outcome effects and probabilities, not the conditional output state. Distinct instruments can share the same effects [12, 36]. This is the obstruction `CE-P5-10`. Likewise, an operational reconstruction does not choose Everett, Bohm, a collapse model, or another interpretation [6, 20, 23]. Paper 5 records this as the open nonclaim `P5-C06`.

# 7 The geometry realization branch

## 7.1 Dependency ladder

The geometry branch starts with a candidate relational interface. It does not start with the assertion that categorical arrows are causal curves. Its reconstruction ladder is

typed events and relation $\longrightarrow$ causal order $\longrightarrow$ conformal class $\longrightarrow$ projective class $\longrightarrow$ clock/scale data $\longrightarrow$ Lorentz metric.

The Hawking-King-McCarthy, Malament, and Levichev results compare Lorentzian spacetimes under specified causality hypotheses [26, 30, 33]. They do not construct a smooth manifold from an arbitrary categorical relation. The EPS program separates null and free-fall data [19, 34]. Clock assumptions and global topology remain visible when a Weyl structure is reduced to a metric one [4].

Connections and curvature enter only after differentiability and nondegeneracy have been supplied. A global coframe is stronger than a Lorentz metric because it trivializes the tangent bundle [22]. Torsion-free projective data do not determine an arbitrary connection. These qualifications appear in `CE-P4-06` and `CE-P4-07`.

**Proposition 7.1** (Geometry branch schema)**.** *Let $G$ be a geometry-branch witness. Suppose $G$ satisfies the exact assumption package `P4-A01` through `P4-A11`; suppose the Paper 1 and Paper 3 inputs are connected to the hypotheses of the selected causal, conformal, projective, clock, and variational theorems by reviewed conversion maps; and suppose every source theorem is used at its stated dimension, regularity, boundary, and field-content scope. Then the corresponding branch conclusions may be composed in that witness.*

*Proof.* This is a typed conditional schema. Each conclusion follows from its named source theorem after the source hypotheses have been discharged. The graph records the required interfaces and assumptions. It does not provide the witness $G$ or the conversion maps. Therefore the registered project claims `P4-C01` through `P4-C04` remain **OPEN**. □

## 7.2 Einstein dynamics

Locality, covariance, and conservation do not uniquely select Einstein's equation. The action

$$S[g] = \int_M \sqrt{|g|}\left(R - 2\Lambda + \alpha R^2\right)\, d^n x, \qquad \alpha \neq 0,$$

is local, diffeomorphism covariant, and variational, but it produces higher-order field equations [7, 39]. This witness makes `P4-C05` **OBSTRUCTED**.

Lovelock's classification applies to a restricted natural tensor class with dimension and derivative-order hypotheses [31, 35]. Boundary terms, extra fields, independent connections, torsion, and nonlocal actions change the problem. The contracted Bianchi identity gives divergence freedom for the geometric tensor, while matter conservation also uses the matter equations and diffeomorphism invariance [27]. Einstein dynamics can be a conditional realization under these restrictions. It is not a consequence of categorical locality alone.

# 8 The quantum realization branch

## 8.1 Two reconstruction routes

Paper 5 retains two finite routes because their assumptions differ. The operational route begins with systems, tests, transformations, and probabilities, then imposes the six principles in the selected CDP theorem [10]. The homogeneous self-dual route begins with finite state-complete ordered models, self-duality, homogeneity, composite assumptions, local tomography, and a qubit [5, 25]. Neither route begins with the Paper 3 outcome labels alone.

The route split matters. Real and quaternionic models survive broad quantum-logical or Hilbert-lattice assumptions [2, 38]. Local tomography and a qubit do not imply complex matrix algebras without the ambient homogeneous self-dual hypotheses. Finite complex C-star algebras may still have direct-sum superselection sectors. These are the obstructions `CE-P5-01`, `CE-P5-04`, `CE-P5-07`, and `CE-P5-08`.

**Proposition 8.1** (Quantum branch schema)**.** *Let $Q$ be a quantum-branch witness. Suppose $Q$ contains an explicit strong-monoidal operational bridge satisfying* `P5-A01` *through* `P5-A03`*. Suppose, in addition, that one complete reconstruction route is selected and every assumption for that route is satisfied. Then the exact conclusion of that route may be applied to $Q$. Born-form probability statements may be added only after the Hilbert or effect structure and the dimension-specific hypotheses of* `P5-A10` *and* `P5-A11` *are available.*

*Proof.* The first implication is the selected imported reconstruction theorem with an explicit input map. The Born step uses Gleason for projection measures in dimension at least three and an all-effects or POVM valuation result for the two-dimensional case [8, 9, 24]. No repository artifact supplies the operational bridge or proves the full route hypotheses. Thus `P5-C01` through `P5-C04` remain **OPEN**. □

## 8.2 Dynamics and measurement

Reversible transformations do not select a time parameter or a one-parameter group. Stone's theorem applies after a strongly continuous unitary group has been chosen [40]. It does not choose the group or its Hamiltonian. This is why the graph retains the dynamics assumptions instead of deriving time evolution from monoidality.

The Born rule is also downstream. Gleason's theorem starts with Hilbert space and a projection measure; it does not reconstruct Hilbert space or complex scalars. Standard Gleason misses dimension two. Paper 5 therefore uses separate dimension-sensitive statements and leaves the proposed master classification `P5-C05` **OBSTRUCTED**.

# 9 The kinetic realization branch

## 9.1 Classical microscopic input

Paper 6 does not consume the quantum interface. It chooses deterministic elastic hard spheres with diameter $\varepsilon$, a grand-canonical initial law, Boltzmann-Grad scaling, and a regular limiting

Boltzmann solution. The main imported source is Deng-Hani-Ma version 3 [16]. Earlier hard-sphere work fixes the historical and analytic comparison [21, 29].

Marginalization is exact and compositional, but it forgets correlations. A product fair-bit law and a perfectly correlated fair-bit law have the same one-particle marginal. This finite nonfaithfulness result is `P6-C01b` and is **PROVED**in Lean. It blocks the inference from a coarse-graining functor to molecular chaos.

The exact rational collision map preserves vector momentum and squared-speed energy for a supplied unit normal. The corresponding Lean statements `P6-C02a` and `P6-C05a` are **PROVED**. The finite Haskell suite checks floating-point collision, marginal, and stochastic-matrix cases; `P6-C02b` is **COMPUTATIONAL**. None of these finite results proves propagation of chaos.

**Theorem 9.1** (Kinetic branch, imported conditional result)**.** *Assume the pinned grand-canonical hard-sphere model and the full package `P6-A01`, `P6-A06` through `P6-A09`, and `P6-A12`. Fix a finite interval on which the regular Boltzmann solution and required Maxwellian bounds exist. For sufficiently small $\varepsilon$, with the threshold chosen after that interval and its bounds, the selected finite rescaled correlations converge in the quantitative $L^1$ mode stated by Deng-Hani-Ma to the factorized Boltzmann correlations, including the hard-core exclusion factor.*

*Proof.* This is the exact imported consequence recorded by `P6-C03`, `P6-C04`, and `P6-C06`. Their status is **CONDITIONAL**. The proof is the cited theorem, not the graph validator, Lean finite model, or Haskell suite. The dependency graph preserves the model, scaling, topology, solution regularity, and quantifier order. □

The theorem does not state convergence at $t = \infty$. It does not cover an arbitrary microscopic interaction or ensemble. Mean-field and Boltzmann-Grad scalings can yield different limit equations. Recollisions require analytic bad-set estimates. Kac's stochastic scalar-velocity model is an executable comparator, not evidence for deterministic hard spheres [28]. These boundaries are recorded by `CE-P6-02`, `CE-P6-03`, `CE-P6-06`, and `CE-P6-09`.

## 9.2 Entropy and solution classes

A static Shannon entropy calculation is not an H theorem. Entropy production requires a dynamics, a collision kernel, a solution class, and enough integrability to justify the sign calculation. Classical regular solutions support the usual local balances under the stated symmetries. The renormalized DiPerna-Lions branch has a different formulation, including an entropy inequality and possible momentum defect [17]. Claims `P6-C05b` and `P6-C05c` are **CONDITIONAL**and deliberately separate. The bundled `P6-C05` remains **OPEN**.

# 10 Three branches, three quantifiers

The logical form of the synthesis is

$$\forall G : \mathsf{Geom}, \quad A_{\mathsf{Geom}}(G) \Longrightarrow C_{\mathsf{Geom}}(G), \tag{2}$$
$$\forall Q : \mathsf{Quant}, \quad A_{\mathsf{Quant}}(Q) \Longrightarrow C_{\mathsf{Quant}}(Q), \tag{3}$$
$$\forall K : \mathsf{Kinetic}, \quad A_{\mathsf{Kinetic}}(K) \Longrightarrow C_{\mathsf{Kinetic}}(K). \tag{4}$$

The three quantified types need not coincide. Equations (2) through (4) do not imply

$$\exists X, \quad A_{\mathsf{Geom}}(X) \wedge A_{\mathsf{Quant}}(X) \wedge A_{\mathsf{Kinetic}}(X).$$

They also do not supply witnesses for any antecedent.

The Lean file `FoundationalReformulation/Synthesis.lean` makes the quantifier distinction explicit. A `SeparateRealizationProgram` has three witness types and three assumption-to-conclusion functions. A `SeparateRealizationTheorems` record stores the three implications. A stronger `JointRealizationProgram` has one witness type, and a `JointRealizationCertificate` must carry one witness satisfying all three assumption packages. Declaring these structures is not a proof that such witnesses exist.

*Warning* 10.1 (No proof by aggregation)*.* Three implications over three types cannot be merged by renaming all three types "physical state." A same-realization theorem needs either literal type identity with compatible assumptions or reviewed conversion maps and a joint satisfiability certificate.

This point also explains the graph's state refinements. The Paper 3 algebraic state, Paper 5 operational state, and Paper 6 kinetic state share a registry word but not a fingerprint. A conversion may be an interesting research project. Until it is specified and reviewed, the graph rejects the substitution.

# 11 Complete obstruction register

The obstruction register gives concrete tests for particular overclaims. Each entry states what must change before a successor theorem can avoid its counterexample. The table includes all 49 live entries.

| ID | Blocked shortcut | Required correction |
|---|---|---|
| `CE-P1-01` | All compatible descent data are effective. | Name the ambient category and an effectiveness theorem. |
| `CE-P1-02` | Composition already supplies locality. | Add covers, separation, and gluing data. |
| `CE-P1-03` | A coverage determines causal order. | Supply an independent causal or realization interface. |
| `CE-P2-01` | Condensation selects physical geometry. | Treat condensation as an ambient mathematical construction. |
| `CE-P2-02` | Full faithfulness holds for every topological space. | Restrict and audit the physical source category. |

| ID | Blocked shortcut | Required correction |
|---|---|---|
| CE-P3-01 | Every monoidal category reconstructs a compact group. | Retain rigidity, symmetry, conjugates, closures, and unit hypotheses. |
| CE-P3-02 | A representation category canonically fixes a group scheme. | Track fibre-functor existence and choice. |
| CE-P3-03 | AQFT reconstructs spacetime by definition. | Treat spacetime-indexed nets as realization comparators. |
| CE-P3-04 | A reconstructed group selects observables. | Supply the observable functor and action separately. |
| CE-P3-05 | Every automorphic action has an invariant state. | Assume amenability or exhibit the state. |
| CE-P3-06 | Symmetry invariance is conservation. | Specify a dynamics, fixed points, and generator domains. |
| CE-P3-07 | Monoidality produces outcomes. | Declare the outcome functor and coherence. |
| CE-P3-08 | A fibre functor is an operational reference frame. | Add a separate physical realization theorem. |
| CE-P3-09 | Labels, effects, and probabilities are one type. | Keep all three interfaces distinct. |
| CE-P4-01 | Causal order fixes a full metric. | Retain conformal freedom and add scale data. |
| CE-P4-02 | Light rays determine free fall. | Supply projective and compatibility data. |
| CE-P4-03 | Local covariance selects Einstein gravity. | Restrict the derivative and variational class. |
| CE-P4-04 | Lovelock uniqueness has no assumptions. | State dimension, order, field, and naturality hypotheses. |
| CE-P4-05 | Causal rigidity constructs spacetime from a bare relation. | Add typed events, smooth structure, metric, and comparison maps. |
| CE-P4-06 | Free-fall paths fix a full connection. | Use torsion-free projective scope or supply torsion observables. |
| CE-P4-07 | A Lorentz metric gives a global coframe. | Use local coframes or prove a global trivialization condition. |
| CE-P4-08 | No second-clock effect gives one global metric. | State clock dynamics, topology, and residual scale. |
| CE-P4-09 | Lovelock proves action uniqueness. | Separate field-equation classification from action equivalence. |
| CE-P4-10 | Bianchi alone proves matter conservation. | Add matter equations and connection assumptions. |
| CE-P4-11 | The GHY term handles every boundary. | Parameterize null segments and joints. |
| CE-P5-01 | Quantum logic forces complex Hilbert space. | Add a complex-scalar selection principle. |
| CE-P5-02 | Standard Gleason covers every dimension. | Handle dimension two separately. |
| CE-P5-03 | Compact closure uniquely characterizes quantum theory. | Distinguish semantics from reconstruction. |
| CE-P5-04 | Compositionality excludes real and quaternionic models. | Add scalar and tomography hypotheses. |
| CE-P5-05 | Categorical scalars are probabilities. | Supply positivity, normalization, effects, and interpretation. |

| ID | Blocked shortcut | Required correction |
|---|---|---|
| CE-P5-06 | Outcome labels canonically define probabilities. | Supply tests, evaluation, and operational equivalence. |
| CE-P5-07 | Tomography and a qubit suffice for complex quantum theory. | Retain the homogeneous self-dual ambient assumptions. |
| CE-P5-08 | Complex selection removes superselection. | Add irreducibility or retain direct-sum sectors. |
| CE-P5-09 | Reversible maps select time and a Hamiltonian. | Choose a continuous one-parameter dynamics first. |
| CE-P5-10 | A POVM fixes state update. | Treat instruments as additional process data. |
| CE-P5-11 | Gleason reconstructs Hilbert space. | Use it only after Hilbert and scalar structure exist. |
| CE-P5-12 | One theorem covers every quantum regime. | Use a route-and-case table. |
| CE-P5-13 | Operational reconstruction solves interpretation. | Keep ontology and collapse as separate additions. |
| CE-P6-01 | Coarse-graining implies molecular chaos. | Prove factorization or propagation of chaos. |
| CE-P6-02 | Large particle number selects Boltzmann dynamics. | Specify interaction and scaling. |
| CE-P6-03 | Collision histories stay independent automatically. | Control recollisions and bad sets analytically. |
| CE-P6-04 | Global PDE existence is a particle derivation. | Separate PDE and microscopic convergence gates. |
| CE-P6-05 | Every rigorous hard-sphere theorem is short-time. | Use current prior art and retain its restrictions. |
| CE-P6-06 | Kac propagation transfers to hard spheres. | Keep the stochastic comparator separate. |
| CE-P6-07 | Static entropy verifies the H theorem. | Supply evolution, solution class, and a sign proof. |
| CE-P6-08 | Marginalization detects independence. | Track correlations through a separate theorem. |
| CE-P6-09 | Arbitrarily long means unconditional all-time. | Preserve finite-horizon quantifier order. |
| CE-P6-10 | Renormalized solutions satisfy classical equalities. | Separate classical and renormalized solution classes. |
| CE-PS-01 | Reviewed branch theorems compose automatically. | Check versions, types, assumptions, reviews, and joint scope. |

Table 3: The complete counterexample and obstruction register.

An obstruction is local to the attacked statement. For example, CE-P4-03 blocks uniqueness without a derivative restriction; it does not refute a correctly scoped Lovelock theorem. CE-P3-05 blocks an unqualified invariant-state claim; it does not refute averaging for compact groups. This local reading is what permits later successor claims to become more precise instead of merely more cautious.

# 12 Evidence and review scope

## 12.1 Evidence classes

The graph keeps eight evidence classes. An imported theorem can support a **CONDITIONAL**claim when its exact version and hypotheses are present. A paper-local or Lean kernel proof can support a **PROVED**claim only at the statement it proves. A computational witness supports a **COMPUTATIONAL**claim on its executed finite domain. An obstruction certificate supports **OBSTRUCTED**. An open record supports honesty about an unmet obligation. Declared interfaces and conditional packages support typing, not existence.

| Evidence class | Valid use | Invalid promotion |
|---|---|---|
| `IMPORTED_THEOREM` | Exact conditional consequence under all source hypotheses | Project proof from citation alone |
| `PAPER_LOCAL_PROOF` | Reviewed local mathematical statement | Stronger stable physical claim |
| `LEAN_KERNEL_PROOF` | Exact checked declaration after dependency audit | Physical theorem encoded as an input field |
| `COMPUTATIONAL_WITNESS` | Finite executed behavior at a named version | Universal or continuum theorem |
| `OBSTRUCTION_CERTIFICATE` | Countermodel to the attacked statement | Refutation of a weaker corrected theorem |
| `DECLARED_INTERFACE` | Typed producer and consumer boundary | Existence or physical adequacy |
| `CONDITIONAL_PACKAGE` | Storage for premises and an implication | Proof of a caller-supplied implication |
| `OPEN_RECORD` | Location of an unmet obligation | Positive premise for a downstream law |

Table 4: Evidence classes preserve the strength of each artifact.

## 12.2 Review freshness

A terminal review is current only for its declared input closure. The six source-paper reviews are bound to exact manuscript bytes at `MANUSCRIPT_ONLY` scope. Their acceptance means that the manuscripts state their results and limits adequately. It does not turn an **OPEN**claim into a theorem.

The Papers 4 through 6 integration closure has separate code and scientific compliance reviews. Both are bound to the exact integration report and carry terminal `APPROVED` verdicts. These reviews support the authoring gate. Task 10's independent Claude code review and the website review are outside this paper's authoring target and remain release gates.

Review scope matters when shared ledgers change. A manuscript-only review can remain current while a source-bundle review becomes stale. The graph does not collapse those answers into one boolean. Each review lists its reviewed inputs and computes a closure digest over their hashes.

# 13 The conditional thesis

The program can now be stated without the usual ambiguity around "emergence." The common pre-geometric layer supplies typed places for contexts, processes, covers, descent, representations, observables, and outcome labels. It does not select a world. A realization branch adds mathematical and physical structure, then invokes theorems whose hypotheses match that structure.

**Theorem 13.1** (Branch-local synthesis)**.** *For each realization branch $B$ in the set*

$$\{\mathsf{Geom}, \mathsf{Quant}, \mathsf{Kinetic}\},$$

*let $X_B$ be a witness of that branch's declared type. If every hard interface input has the registered revision, every branch assumption is satisfied, every conversion is explicit and reviewed, and every imported theorem is used at its exact scope, then the verified or conditional conclusions in branch $B$ may be composed for $X_B$.*

*Proof.* Apply the admissibility predicate (1) separately to each claim in the branch closure. Type compatibility permits composition along hard edges. Acyclicity gives an order in which premises can be checked. Assumption preservation and review freshness prevent a claim from entering the target at a stronger version than its evidence. The result is one implication per branch. The proof does not identify $X_{\mathsf{Geom}}$, $X_{\mathsf{Quant}}$, and $X_{\mathsf{Kinetic}}$. □

The theorem is a statement about admissible composition. It does not prove `PS-C02`, because the repository has not supplied all branch witnesses and bridge maps. It also does not prove `PS-C01` or `PS-C03`; the current graph is an authoring artifact whose complete physical adequacy still requires independent review. All four synthesis claims remain **OPEN**.

The strongest scientific thesis supported here is:

> Categorical locality and representation theory provide a disciplined order for stating physical reconstruction problems. Lorentzian geometry, finite complex quantum theory, and the selected Boltzmann limit can occur as separate realizations when their branch-specific hypotheses, conversions, and source theorems are supplied. The present artifacts do not show that these laws are unique consequences of the common pre-geometric layer.

This formulation is weaker than a universal foundation. Its benefit is specificity: it locates the work required for a stronger theorem. A future same-realization claim needs a joint witness type, satisfiable combined assumptions, reviewed conversion maps among state notions, and evidence that the branch laws refer to that one witness. A future necessity claim needs a classification theorem showing that alternatives are excluded. Neither can be replaced by a larger diagram.

# 14 Reproducibility contract

The synthesis has four independent checks.

1. The graph suite runs 16 positive and adversarial tests. The live graph validator checks the authoring target and the TeX claim projection.
2. Lean builds the repository and checks the separate-realization records. A forbidden-token scan rejects `sorry` and `admit`.
3. Haskell builds with `-Wall -Wcompat -Werror`, runs the full test suite, and executes the demonstration program.
4. LaTeX compiles the manuscript. The PDF page gate, character audit, and humanizer audit apply to the exact reviewed source.

The checks answer different questions. A successful graph gate shows that the finite dependency object is closed and current. Lean checks exact formal declarations. Haskell checks executable finite behavior. LaTeX checks that the manuscript builds. None substitutes for an analytic theorem or a physical realization.

# A Interface inventory

| Interface | Branch | Availability | Content |
|---|---|---|---|
| `P1-I-CTX-SIGNATURE` | shared | declared | Parameterized contexts, processes, and composition. |
| `P1-I-COVER-SIGNATURE` | shared | declared | Cover-family signature without a selected physical coverage. |
| `P1-I-DESCENT-SIGNATURE` | shared | declared | Descent data and a type for a supplied effectiveness witness. |
| `P2-I-CONDENSED-REALIZATION` | shared | open | Optional condensed realization; every use needs a receipt. |
| `P3-I-OBSERVABLE` | shared | open | Context-indexed observable data and symmetry action. |
| `P3-I-OUTCOME` | shared | open | Typed pre-geometric outcome labels without probabilities. |
| `P4-I-RELATIONAL-CANDIDATE` | geometry | open | Candidate causal relation in a geometry realization. |
| `P4-I-LORENTZIAN` | geometry | open | Lorentzian metric package with conformal and projective data. |
| `P5-I-OPERATIONAL` | quantum | open | Strong-monoidal operational-probabilistic bridge. |
| `P5-I-QUANTUM` | quantum | open | Finite complex quantum realization output. |
| `P6-I-MICROSTATE` | kinetic | open | Classical many-body and grand-canonical microscopic data. |
| `P6-I-KINETIC` | kinetic | conditional | Pinned hard-sphere kinetic-limit output. |
| `PS-I-DEPENDENCY-GRAPH` | shared | open | Content-addressed branch dependency closure. |

Table 5: All typed interfaces in the graph.

## B Excluded and prospective edges

**Lorentzian realization to Paper 3 measurement.** The edge

$$\texttt{P4-I-LORENTZIAN} \dashrightarrow \texttt{P3-C05}$$

is prospective. It represents the desired comparison between pre-geometric outcomes and localized measurements. It is excluded from the hard closure because Paper 4 already consumes Paper 3 interfaces. A hard return edge would create a reconstruction cycle.

**Condensed receipts.** The edges from `P2-I-CONDENSED-REALIZATION` to the geometry, quantum, and kinetic branches are prospective. A successor graph may make one hard only after recording a receipt with exact preserved operations and a current review.

**Quantum to kinetic.** No edge connects `P5-I-QUANTUM` to the kinetic branch. The absence is intentional. The kinetic theorem starts with a classical hard-sphere law, whereas the quantum interface concerns operational or Hilbert-space states. No exact conversion theorem is registered.

## C Release boundary

This manuscript completes the synthesis authoring target only after its own bounded AGY review is current. The repository release remains a separate target. It requires the independent code reviews, website implementation, responsive and mathematical rendering checks, deployed URL verification, and final publication steps specified in the project plan. Those tasks cannot strengthen a scientific status. They can only confirm that the current statuses and artifacts are represented correctly.

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