Representation-Theoretic Emergence of Measurement, Symmetry, and Observable Structure

Matthew Long
The YonedaAI Collaboration
YonedaAI Research Collective
Chicago, IL
matthew@yonedaai.com · https://yonedaai.com

23 Jul 2026

Abstract

This paper asks what representation theory can reconstruct before one has spacetime, a Hilbert-space measurement model, or probabilities. The main mathematical branch is the compact-group duality of Doplicher and Roberts. Its input is a strict symmetric monoidal C-star category with conjugates, subobjects, direct sums, and complex scalar endomorphisms of the tensor unit. Its output is a compact group, unique up to isomorphism, whose category of finite-dimensional continuous unitary representations is isomorphic to the input category. This is an imported theorem, not a proof contributed by this project. It needs no separately supplied fibre functor and it does not reconstruct an affine group scheme, a supergroup, an observable algebra, a state, or a measurement.

The missing bridges are made explicit. A context-indexed observable functor and a continuous natural action of the reconstructed group are separate data. For an exhibited state and a compact action, Haar averaging gives an invariant state objectwise. The finite version is proved in the accompanying Lean interface and tested in Haskell. Neither result selects a preferred state nor produces a natural family across contexts. Indeed, a nonamenable translation action supplies an obstruction to unconditional invariant-state existence, while locally covariant quantum field theory supplies a separate warning against natural state selection.

Symmetry invariance is also kept distinct from conservation. Conservation refers to a separately declared one-parameter dynamics and, where used, its generator domain. A strong monoidal functor to finite sets supplies only compositional outcome labels. It supplies no effects, scalar evaluation, normalization, Born rule, or operational probability. Fibre-functor comparison torsors are called algebraic frame data; physical reference frames require a later realization. Haag-Kastler and locally covariant quantum field theory are treated only as comparators that already presuppose spacetime regions or spacetimes.

Consequently, registered claims `P3-C01`, `P3-C02`, `P3-C04`, and `P3-C05` remain **OPEN**. The bundled state-and-conservation claim `P3-C03` is **OBSTRUCTED**. All assumptions `P3-A01` through `P3-A09` remain **UNVALIDATED**.

Contents

1 Question, answer, and claim discipline

1.1 The research question

The guiding question is:

> Starting from the representation-bearing context interfaces proposed in Papers 1 and 2, which symmetry, observable, state, outcome, and frame structures follow by theorem, and which must be supplied as independent typed bridges?

The short answer is that considerably less follows than informal phrases such as "symmetry produces observables" or "measurement emerges from representations" suggest. An exact compact-group reconstruction is available on a stringent C-star-categorical branch. The remaining physical structures need additional interfaces. The construction exposes those interfaces.

The word "emergence" in the title therefore means a dependency-controlled reconstruction program. It does not mean that every object in the title has already been derived. The compact group is the only item supplied by the selected imported reconstruction theorem. Observable algebras, actions, states, dynamics, outcome labels, frame data, and physical realization maps occupy different types.

1.2 Registered claims

The first claim remains open even though an imported theorem is available. The repository has not constructed a category satisfying its hypotheses from the Paper 1 and Paper 2 interfaces. The registered wording also mentions a fibre functor. That datum belongs to the neutral Tannakian comparator, not to the selected Doplicher-Roberts theorem. This paper corrects the branch structure without rewriting distinct theorems as one theorem.

The second claim remains open because a group does not choose the algebra on which it acts. The third is obstructed because neither invariant states nor conservation follow in the registered generality, and because the two conclusions require different data. The fourth remains open because an outcome functor must still be constructed for the selected physical contexts. The fifth remains open until later papers supply spacetime and an adequacy theorem for physical measurement.

1.3 Assumption ledger

Claim	Registered target	Status
P3-C01	The selected representation category and fibre functor reconstruct a symmetry object unique up to the stated equivalence.	**OPEN**
P3-C02	The reconstructed symmetry data act on a declared observable algebra/interface compatible with context composition.	**OPEN**
P3-C03	Under positivity, continuity, and normalization assumptions, the observable interface admits invariant states and conserved quantities with a proved correspondence.	**OBSTRUCTED**
P3-C04	Monoidal composition produces typed pre-geometric outcome objects while probability remains an independent interface.	**OPEN**
P3-C05	A realization map cleanly separates pre-geometric outcome data from ordinary measurements localized in a realized spacetime.	**OPEN**

Table 1: Paper 3 claim status. The exact wording is inherited from the claim register. The compact averaging result below is narrower than the bundled claim P3-C03.

ID	Required content	Status
P3-A01	The main-line category is a strict symmetric monoidal C-star category with conjugates.	**UNVALIDATED**
P3-A02	A faithful exact tensor fibre functor exists when the neutral Tannakian comparator is invoked. It is not attached to the main DR theorem.	**UNVALIDATED**
P3-A03	Subobjects, direct sums, scalar unit endomorphisms, and the remaining branch-specific reconstruction hypotheses hold.	**UNVALIDATED**
P3-A04	A context-indexed unital C-star or star-algebra functor of observables is separately declared or constructed.	**UNVALIDATED**
P3-A05	A continuous action by unital star-automorphisms is natural in context and equivariant with the comparison maps used.	**UNVALIDATED**
P3-A06	Every state used is exhibited as a positive normalized functional, while a compatible family carries an additional naturality proof.	**UNVALIDATED**
P3-A07	Averaging continuity is supplied, and any conservation statement uses a separately declared one-parameter dynamics and generator domain.	**UNVALIDATED**
P3-A08	A separately declared strong monoidal finite outcome-label functor has no evaluation into ordered normalized scalars.	**UNVALIDATED**
P3-A09	Later spacetime, algebra, and physical-outcome realization maps are declared without identifying labels with measurements.	**UNVALIDATED**

ID	Required content	Status

Table 2: All Paper 3 assumptions remain unvalidated.

2 Input boundary from Papers 1 and 2

2.1 What may be imported

Paper 1 proposes a category Ctx of physical contexts, a class of processes, a symmetric monoidal product, coverage data, and coefficient assignments. Paper 2 proposes controlled condensed realizations for selected topological and additive fields. Their registered reconstruction claims remain open. Paper 3 may therefore use their signatures as declared interfaces, but it may not cite them as established physical categories or established universal realization theorems.

For the present paper, let

$$(\mathsf{Ctx}, \otimes, I)$$

denote a small symmetric monoidal category supplied as input. No object of Ctx is assumed to be a spacetime. No arrow is assumed to be time evolution. No tensor product is assumed to be spatial separation. No cover is assumed to be an open cover. The symbols keep their Paper 1 types and nothing more.

The condensed layer of Paper 2 is optional here. When a coefficient object already carries a controlled condensed realization, a later observable functor may be asked to respect it. No result in this paper needs that compatibility. In particular, a condensed realization does not create a C-star norm, positivity cone, state space, time evolution, or operational probability.

2.2 The representation-bearing input

The compact reconstruction branch uses a category

$$\mathcal{C}_{\mathrm{rep}}$$

that is not automatically identical to Ctx. It may be attached globally to the theory, attached to a selected context, or obtained through another construction. Those possibilities have different variance and coherence obligations. Paper 3 leaves the attachment map open.

The distinction matters. The source of Doplicher-Roberts duality is already a representation-like C-star category. It is not a category of bare contexts. A complete project proof would need a construction

$$\mathfrak{R} : \mathsf{Ctx} \longrightarrow \{\text{admissible C-star tensor categories}\},$$

or a comparably explicit global assignment, followed by verification of all DR hypotheses. Neither map nor verification is presently available.

2.3 Frozen input record

The minimal record used for the rest of the paper contains:

1. a small symmetric monoidal context category $(\mathsf{Ctx}, \otimes, I)$;
2. a candidate strict symmetric monoidal C-star category $\mathcal{C}_{\mathrm{rep}}$;
3. proposed conjugates, subobjects, direct sums, and an identification $\mathrm{End}(\mathbf{1}) \cong \mathbb{C}$;
4. a later observable functor $\mathcal{A} : \mathsf{Ctx} \to \mathsf{uCStar}$ or $\mathsf{Ctx} \to \mathsf{StarAlg}$;
5. a later action α of a reconstructed group on $\mathcal{A}$;
6. any states to be averaged, supplied explicitly;

7. a separate one-parameter dynamics τ where conservation is discussed;
8. a separate outcome-label functor $\mathcal{O} : \mathsf{Ctx} \to \mathsf{FinSet}$;
9. algebraic frame and later physical realization data.

Items four through nine do not appear in the source type of the selected compact-group reconstruction theorem. Listing them in the same record does not make them consequences of the first three items.

3 Three reconstruction branches that must not be merged

3.1 The selected compact-group branch

Theorem 3.1 (Imported Doplicher-Roberts compact duality)**.** *Relative to registered claim `P3-C01` and assumptions `P3-A01` and `P3-A03`, let $\mathcal{C}_{\mathrm{rep}}$ be a strict symmetric monoidal C-star category with conjugates, subobjects, direct sums, and $\mathrm{End}(\mathbf{1}) = \mathbb{C}$. Then the official source summary states that $\mathcal{C}_{\mathrm{rep}}$ is isomorphic to a category $H(G)$ of finite-dimensional continuous unitary representations of a compact group G, unique up to isomorphism.*

This is the theorem stated in the official Springer summary of [10]. The source audit did not obtain the full text at a theorem-number locator. Accordingly, this paper neither invents such a locator nor claims an internal proof. Every adjective in the theorem is an input.

The theorem does not mention a separately selected fibre functor. It reconstructs a compact topological group, not an affine group scheme. It does not reconstruct a supergroup. It does not return a functor $\mathcal{A} : \mathsf{Ctx} \to \mathsf{uCStar}$. It does not return states, dynamics, outcomes, frames, or spacetime.

Strictness also deserves care. Ordinary coherence can often replace a monoidal category by an equivalent strict one, but such a replacement must preserve the C-star structure, symmetry, conjugates, subobjects, direct sums, and unit endomorphisms relevant to the theorem. Paper 3 records strictness as part of `P3-A01` rather than silently performing that transfer.

3.2 Neutral Tannakian comparison

Let k be a field. The neutral theorem of Deligne and Milne starts with a rigid abelian k-linear tensor category $\mathcal{T}$ satisfying

$$\mathrm{End}(\mathbf{1}) = k$$

and an exact faithful k-linear tensor functor

$$\omega : \mathcal{T} \longrightarrow \mathsf{Vec}_k.$$

It reconstructs the affine group scheme

$$\mathrm{Aut}^{\otimes}(\omega)$$

and identifies $\mathcal{T}$ with its finite-dimensional representations [9, Theorem 2.11].

This branch uses `P3-A02`. It does not replace `P3-A01`. Its group scheme is not the compact group of theorem 3.1. The fibre functor is part of the reconstruction datum. Removing rigidity weakens the output to an affine monoid scheme, as the proof conclusion in the corrected source makes explicit.

3.3 Nonneutral Tannakian comparison

When a base-field-valued fibre functor is unavailable, the intrinsic object is not a secretly chosen neutral group. Deligne and Milne describe a gerbe of fibre functors

$$\mathsf{FIB}(\mathcal{T}).$$

Fibre functors after base change and their tensor isomorphisms form this nonneutral object [9, Definition 3.7 and Theorem 3.9].

If ω and η are two fibre functors, their tensor comparison object

$$\mathrm{Isom}^{\otimes}(\omega, \eta)$$

is a torsor under the corresponding automorphism group [9, Theorem 3.2]. This fact will later motivate algebraic frame data. It does not make either fibre functor a physical reference frame.

3.4 Super-Tannakian comparison

Deligne's super-Tannakian theorem has a third source type. Over an algebraically closed field of characteristic zero, a finitely tensor-generated tensor category meeting the stated Schur-finiteness condition can be equivalent to

$$\mathsf{Rep}(G, \epsilon),$$

where G is an affine supergroup scheme and ϵ is parity data [8]. The output is neither the compact group of Doplicher-Roberts nor an ordinary affine group scheme with the parity forgotten.

Branch	Characteristic input	Output
Doplicher-Roberts	Strict symmetric monoidal C-star category, conjugates, subobjects, direct sums, $\mathrm{End}(\mathbf{1}) = \mathbb{C}$	Compact group G and category $H(G)$
Neutral Tannakian	Rigid abelian k-linear tensor category plus exact faithful fibre functor	Affine group scheme $\mathrm{Aut}^{\otimes}(\omega)$
Nonneutral Tannakian	Tannakian category considered without a chosen neutralization	Affine gerbe of fibre functors
Super-Tannakian	Characteristic-zero tensor category plus Schur-finiteness hypotheses	Affine supergroup scheme with parity datum

Table 3: The branches have different source types and different codomains. Their hypotheses and conclusions cannot be intersected into a synthetic theorem.

3.5 Status consequence

The imported theorem makes `P3-C01` mathematically plausible on the chosen branch. It does not close the claim. No repository construction currently shows that the physical representation category satisfies `P3-A01` and `P3-A03`. The fibre-functor phrase in the registered claim belongs only to the neutral comparator governed by `P3-A02`. Thus the honest status remains **OPEN**.

4 The observable bridge is separate data

4.1 Observable functor

Assume a functor

$$\mathcal{A} : \mathsf{Ctx} \longrightarrow \mathsf{uCStar}, \qquad c \longmapsto \mathcal{A}(c),$$

where morphisms are faithful unital star-homomorphisms, or use a precisely declared star-algebra target if norm completeness is not available. This is assumption `P3-A04`. The chosen target affects every later claim about positivity, continuity, integration, and state spaces.

For a process $f : c \to d$, write

$$\mathcal{A}(f) : \mathcal{A}(c) \longrightarrow \mathcal{A}(d).$$

Covariance is a convention, not a theorem. Another model may use contravariance. What matters is that the variance be fixed before state compatibility is stated.

The functor $\mathcal{A}$ is not returned by theorem 3.1. Indeed, for any compact group G and any unital C-star algebra B, the trivial action

$$\alpha_g = \mathrm{id}_B$$

exists. The same group can act nontrivially on many inequivalent algebras. The group therefore cannot select B from its own abstract group structure.

4.2 Natural group action

Let G be the compact group reconstructed on the DR branch. A context-compatible action is a family

$$\alpha_{g,c} \in \mathrm{Aut}(\mathcal{A}(c))$$

such that

$$\alpha_{e,c} = \mathrm{id}, \qquad \alpha_{gh,c} = \alpha_{g,c} \circ \alpha_{h,c},$$

and for every $f : c \to d$,

$$\mathcal{A}(f) \circ \alpha_{g,c} = \alpha_{g,d} \circ \mathcal{A}(f). \tag{4.1}$$

Point-norm continuity means that

$$g \longmapsto \alpha_{g,c}(a)$$

is norm continuous for each c and $a \in \mathcal{A}(c)$. These are the contents of `P3-A05`.

If $\mathcal{A}$ is monoidal, its comparison maps

$$m_{c,d} : \mathcal{A}(c) \otimes \mathcal{A}(d) \longrightarrow \mathcal{A}(c \otimes d)$$

must also be declared. Equivariance means

$$m_{c,d} \circ (\alpha_{g,c} \otimes \alpha_{g,d}) = \alpha_{g,c\otimes d} \circ m_{c,d}.$$

No such equation follows merely from reconstructing G.

Proposition 4.1 (Observable nonselection)**.** *For registered claim `P3-C02` under assumptions `P3-A04` and `P3-A05`, the abstract compact group G alone does not determine a context-indexed observable functor $\mathcal{A}$ or a natural action α on it.*

Proof. Every unital C-star algebra admits the trivial G-action. Choose two nonisomorphic unital C-star algebras, for example $\mathbb{C}$ and $C([0,1])$. Both carry that action, so the group cannot distinguish them. A single algebra such as $C(G)$ can carry both a trivial action and a translation action. Hence neither the algebra nor its action is selected by the abstract group. Functorial assignment across `Ctx` adds still more data. □

The proposition is a negative typing result. It does not say that a particular physical construction cannot supply $\mathcal{A}$ and α. It says only that the DR output is insufficient. Accordingly, `P3-C02` remains **OPEN**.

4.3 Reverse-direction AQFT comparators

Doplicher and Roberts also studied the reconstruction of gauge and field structure from a C-star algebra together with a selected endomorphism semigroup/category, permutation symmetry, conjugates, and physical superselection input [11, 12]. These results begin on the observable side. They are evidence that a rich observable interface can lead to gauge structure. They do not reverse automatically into a construction of observables from the bare group of theorem 3.1.

The access boundary is material. For both reverse-direction sources, the present source audit checked official abstract-level statements but did not pin the full theorem hypotheses at a detailed locator. This paper therefore uses them as comparators and does not quote a theorem that the audit did not inspect.

5 States, averaging, and the obstruction

5.1 States are exhibited inputs

For a unital C-star algebra A, a state is a complex-linear functional

$$\omega : A \to \mathbb{C}$$

that is positive and normalized:

$$\omega(a^*a) \geq 0, \qquad \omega(1) = 1.$$

Whenever this paper averages a state, that state is included in the input. No state is inferred from a representation label or outcome label. This is the objectwise part of `P3-A06`.

For $f : c \to d$ and the covariant algebra convention above, a compatible contravariant family satisfies

$$\omega_c = \omega_d \circ \mathcal{A}(f). \tag{5.1}$$

Objectwise choices do not establish this equation. Even objectwise invariant states do not establish it. Naturality is a separate property of the whole family.

5.2 Compact averaging

Let G be compact with normalized Haar measure dg, and let $\alpha : G \to \mathrm{Aut}(A)$ be point-norm continuous. Define

$$E(a) = \int_G \alpha_g(a)\, dg. \tag{5.2}$$

The integral is a norm-valued integral. Point-norm continuity and compactness make it well defined.

Proposition 5.1 (Compact averaging at one algebra)**.** *For registered claim `P3-C03` under assumptions `P3-A05`, `P3-A06`, and the compactness and point-norm-continuity part of `P3-A07`, let A be a unital C-star algebra, let G act on A by point-norm-continuous unital star-automorphisms, and let ω be an exhibited state. Then E in* (5.2) *is unital and positive, and*

$$\overline{\omega} = \omega \circ E$$

is a G-invariant state.

Proof. Unitality follows from $\alpha_g(1) = 1$ and normalization of Haar measure. If $a \geq 0$, then every $\alpha_g(a) \geq 0$ because star-automorphisms preserve the positive cone. The norm integral of this continuous positive-valued function remains positive, so E is positive. Consequently $\omega \circ E$ is positive and

$$(\omega \circ E)(1) = \omega(1) = 1.$$

Normalized Haar measure on a compact group is bi-invariant. For $h \in G$, its right invariance gives

$$E(\alpha_h(a)) = \int_G \alpha_{gh}(a)\, dg = \int_G \alpha_g(a)\, dg = E(a).$$

Thus $\overline{\omega} \circ \alpha_h = \overline{\omega}$. □

The same calculation makes E idempotent and gives

$$\mathrm{ran}(E) = A^G := \{a \in A \mid \alpha_g(a) = a \text{ for all } g \in G\}.$$

Those familiar facts do not select the initial ω. Nor do they prove that a collection $\{\overline{\omega}_c\}_{c \in \mathsf{Ctx}}$ satisfies (5.1).

The source [14] concerns a stronger compact ergodic setting and a unique invariant state that is a trace. Theorem 5.1 does not assume ergodicity and makes no uniqueness claim. Its proof is the ordinary Haar-averaging argument rather than an invocation of that stronger theorem.

5.3 Finite executable form

For a finite group G, normalized Haar integration becomes

$$E(a) = \frac{1}{|G|} \sum_{g \in G} \alpha_g(a). \tag{5.3}$$

The Lean layer models a finite normalized state by rational weights, permutation actions on finite types, invariant observables, equivariant maps, and the average in (5.3). It proves normalization of the averaged state and invariance under the finite action.

Theorem 5.2 (Finite averaging identity)**.** *For the finite witness associated with registered claim `P3-C03`, under the finite analogues of assumptions `P3-A05`, `P3-A06`, and `P3-A07`, let a finite group act by permutations on a finite sample type and let w be a normalized nonnegative rational weight function. Then*

$$\overline{w}(x) = \frac{1}{|G|} \sum_{g \in G} w(g^{-1}x)$$

is normalized and invariant.

Proof. Summing first over x and then over g, every permutation $x \mapsto g^{-1}x$ reindexes the finite sum:

$$\sum_x \overline{w}(x) = \frac{1}{|G|} \sum_g \sum_x w(g^{-1}x) = \frac{1}{|G|} \sum_g 1 = 1.$$

For $h \in G$, the substitution $g \mapsto h^{-1}g$ permutes the finite group, so $\overline{w}(h^{-1}x) = \overline{w}(x)$. The repository theorem `averageFiniteState_invariant` kernel-checks the invariance reindexing, while the constructor proof checks normalization. □

This finite theorem is genuinely proved at its stated scope. It is not a formalization of C-star positivity, Haar integration, amenability, or natural state families. The companion Haskell tests are computational evidence only.

5.4 Why arbitrary actions need not have invariant states

Let F_2 be the free group on two generators. It acts by left translation on the unital commutative C-star algebra

$$\ell^\infty(F_2).$$

An invariant state for this action is exactly a left-invariant mean. The group is nonamenable, so no such mean exists. Classical invariant-mean and fixed-point boundaries are discussed in [6, 7, 17].

This counterexample is registered as `CE-P3-05`. It refutes the implication

$$\text{unital C-star algebra} + \text{automorphic group action} \implies \text{invariant state}.$$

Compactness or an appropriate amenability hypothesis does real work. An exhibited fixed state is another legitimate input.

There is also a nonunital warning. Translation of $\mathbb{R}$ on $C_0(\mathbb{R})$ has no translation-invariant state corresponding to a finite invariant probability measure, even though $\mathbb{R}$ is amenable. The compact-convex argument for a unital state space cannot be transferred without checking the changed hypotheses.

5.5 No automatic natural family

Brunetti, Fredenhagen, and Verch define a state as a positive normalized functional, then explain why one cannot generally select a physically good state naturally under every spacetime embedding [4, pp. 15–16]. Their remedy is a contravariant assignment of state spaces rather than a single preferred natural state.

This is not a pre-geometric theorem. BFV already starts with four-dimensional oriented, time-oriented globally hyperbolic Lorentzian spacetimes. It is nevertheless an exact warning against moving from objectwise existence to a canonical natural family.

5.6 Why `P3-C03` is obstructed

Claim `P3-C03` bundles invariant states and conserved quantities with a proved correspondence. The first part fails for arbitrary actions by `CE-P3-05`. The natural-family reading fails without additional compatibility, with BFV providing a physical comparator. The conservation part is not even typed until a dynamics has been declared. For those reasons the exact registered claim is **OBSTRUCTED**, not merely open.

6 Symmetry invariance is not conservation

6.1 Two actions

The symmetry action is

$$\alpha : G \longrightarrow \mathrm{Aut}(A).$$

A dynamics is separately declared as

$$\tau : \mathbb{R} \longrightarrow \mathrm{Aut}(A).$$

The two actions may commute:

$$\tau_t \circ \alpha_g = \alpha_g \circ \tau_t.$$

Commutation still does not identify them. The parameter g labels symmetry transformation, while t labels a chosen one-parameter evolution.

An observable a is symmetry fixed when

$$\alpha_g(a) = a \quad \text{for every } g \in G.$$

It is dynamically conserved when

$$\tau_t(a) = a \quad \text{for every } t \in \mathbb{R}.$$

Neither predicate implies the other for unrelated actions. This is counterexample `CE-P3-06`.

6.2 Generator and domain

For a point-norm-continuous C-star dynamics, define the generally unbounded generator on

$$D(\delta) = \left\{ a \in A \;\middle|\; \lim_{t\to 0} \frac{\tau_t(a) - a}{t} \text{ exists in norm} \right\}$$

by

$$\delta(a) = \lim_{t\to 0} \frac{\tau_t(a) - a}{t}.$$

The domain is part of the statement. Writing $\delta(a)$ for an arbitrary $a \in A$ would be ill typed.

Proposition 6.1 (Fixed points and the declared generator)**.** *For the conservation component of registered claim `P3-C03`, under assumption `P3-A07`, let τ be the separately declared point-norm-continuous one-parameter automorphism group with generator δ. For $a \in D(\delta)$,*

$$\tau_t(a) = a \textit{ for every } t \quad \Longleftrightarrow \quad \delta(a) = 0.$$

Proof. If $\tau_t(a) = a$ for every t, the difference quotient vanishes, hence $\delta(a) = 0$. Conversely, the orbit $t \mapsto \tau_t(a)$ remains in $D(\delta)$ and has derivative

$$\frac{d}{dt}\tau_t(a) = \tau_t(\delta(a)) = 0.$$

The orbit is therefore constant and equals its value a at $t = 0$. □

This proposition is a dynamics fixed-point statement. It is not Noether's theorem. It contains no current, charge density, continuity equation, stress tensor, or spacetime support.

6.3 Stone's theorem is a different layer

If the system has already been represented on a Hilbert space and one has a strongly continuous unitary group

$$U_t = e^{itH},$$

Stone's theorem supplies a self-adjoint generator [15]. That result adds a Hilbert representation, strong continuity, and an unbounded operator domain. None follows from the monoidal structure of Ctx or from theorem 3.1.

Paper 3 therefore does not write a Hamiltonian into the primitive interface. A later realization may relate the algebraic dynamics τ to a unitary implementation, but the relation and its hypotheses must be stated.

6.4 Noether is later and stronger

Buchholz, Doplicher, and Longo prove a weak AQFT Noether result under a Minkowski local-net setting, a Hilbert-space realization, unitary symmetry implementation, vacuum and localization properties, and the split property [5]. Their paper also notes the failure of bounded-region local implementers in some models and leaves reconstruction of currents from local generators as a major problem.

That source is useful here because its assumptions are explicit. It does not support a pre-geometric theorem from an abstract compact symmetry to a conserved current. Paper 3 makes no such claim.

The physical quantum-frame review of [3] gives another useful warning: group-induced superselection restrictions are not equivalent to conservation laws. Its setting already contains Hilbert spaces, unitary representations, density operators, operations, POVMs, and probabilities, so it remains a comparator rather than an input.

7 Probability-free outcome labels

7.1 The minimal interface

Declare a functor

$$\mathcal{O} : \mathsf{Ctx} \longrightarrow \mathsf{FinSet}.$$

For contexts c, d, declare coherent bijections

$$\mu_{c,d} : \mathcal{O}(c) \times \mathcal{O}(d) \xrightarrow{\cong} \mathcal{O}(c \otimes d)$$

and a unit bijection

$$u : \{*\} \xrightarrow{\cong} \mathcal{O}(I).$$

Naturality, associativity, symmetry, and unit diagrams make $\mathcal{O}$ a strong symmetric monoidal functor. This is the exact content requested by `P3-A08`.

An element

$$o \in \mathcal{O}(c)$$

is an outcome label. It need not be an idempotent, projector, effect, event, proposition, sample point, numerical value, or probability. The target FinSet records a finite typed collection and compositional relabeling.

Proposition 7.1 (Monoidal label composition)**.** *For registered claim `P3-C04` under assumption `P3-A08`, a declared strong monoidal outcome-label functor $\mathcal{O} : \mathsf{Ctx} \to \mathsf{FinSet}$ supplies associative and unital composition of labels, but it supplies no scalar evaluation or probability map.*

Proof. The maps $\mu_{c,d}$ and u, together with the strong monoidal coherence diagrams, provide label composition and its associativity and unit laws. The signature contains no morphism with codomain $[0, 1]$, no ordered scalar object, no normalization axiom, and no map from labels to effects. Therefore no scalar evaluation or probability map can be projected from the declared fields. $\square$

This proposition is syntactic at the stated signature. It does not prove physical adequacy of the labels. It also does not prove that a suitable $\mathcal{O}$ exists for the physical contexts. Thus `P3-C04` remains **OPEN**.

7.2 Effects and tests are separate

If $\mathcal{A}(c)$ is a unital C-star algebra, one may later define

$$\mathrm{Eff}(c) = \{e \in \mathcal{A}(c) \mid 0 \le e \le 1\}.$$

A finite test might then be a typed family of effects satisfying some declared sum or compatibility law. Neither type is present in $\mathcal{O}$.

Even after a state ω and an effect e have been supplied, the scalar

$$\omega(e)$$

is only an algebraic evaluation until an operational interpretation is stated. Calling it a physical probability requires a measurement semantics and calibration assumptions that belong to Paper 5. Calling it the Born rule would require still more structure.

Abramsky and Heunen explicitly separate preparations, measurements, finite outcome labels, and an evaluation map [2]. Their tensor-product evaluation law already expresses weighted or probabilistic content. The safe lesson for Paper 3 is that labels and evaluation are separate interfaces.

Abramsky and Coecke provide an important categorical quantum comparator [1]. Their strong compact closure and biproduct structure supports branching, scalars, and an abstract Born-rule discussion. That richer semantics is absent from the minimal probability-free interface here.

7.3 Executable outcome composition

The Lean interface defines a phantom-typed label wrapper

$$\mathsf{OutcomeLabel}(c)$$

and concatenates finite words when contexts compose. The phantom type prevents a label for one context type from being silently used as a label for another. It contains no state, effect, evaluator, real number, or normalization field.

The Haskell interface follows the same design. Its tests check composition, associativity at the stored-word level, and unit behavior. Those tests are **COMPUTATIONAL**. They do not establish the existence or physical interpretation demanded by `P3-C04`.

8 Algebraic frame data

8.1 Fibre-functor comparisons

On the neutral Tannakian comparator, suppose

$$\omega, \eta : \mathcal{T} \longrightarrow \mathsf{Vec}_k$$

are fibre functors. The tensor isomorphism object

$$\mathrm{Isom}^{\otimes}(\omega, \eta)$$

is a torsor under $\mathrm{Aut}^{\otimes}(\omega)$ [9, Theorem 3.2]. A selected point after suitable base change trivializes the torsor and compares the two neutralizations.

This motivates the following terminology:

> An *algebraic frame datum* is a declared fibre-functor comparison torsor together with whatever base change and trivialization have actually been supplied.

The definition is intentionally modest. It does not assert that an agent holds the frame, that rods or clocks have been constructed, that a coordinate system exists, or that a quantum measurement has been calibrated. Those interpretations need a later theorem.

8.2 Frame conflation counterexample

A fibre functor is an exact faithful tensor functor into a linear target. A physical quantum reference frame in [3] begins with Hilbert-space and probabilistic structure. The source and target types are different. No renaming makes them equal.

This is registered counterexample `CE-P3-08`. Its lesson is not that torsors are useless. Torsors are the correct algebraic language for comparing neutralizations. The lesson is that an algebraic comparison requires a realization map before it carries operational frame meaning.

8.3 The main branch needs no frame choice

The selected DR theorem does not ask for a fibre functor. Therefore an algebraic frame datum is not silently inserted into theorem 3.1. Frame comparisons are presented only on the Tannakian comparator branch. This separation avoids using a neutral theorem's input to strengthen the compact C-star theorem's conclusion.

9 Later realization into spacetime measurement

9.1 Separate realization maps

Suppose Paper 4 later constructs a category of realized spacetimes Loc and a functor

$$R : \mathsf{Ctx} \longrightarrow \mathsf{Loc}.$$

Suppose also that one has a spacetime-indexed observable theory

$$\mathcal{B} : \mathsf{Loc} \longrightarrow \mathsf{uCStar}.$$

A comparison between pre-geometric observables and realized observables is then additional data, for example a natural transformation

$$\iota : \mathcal{A} \Longrightarrow \mathcal{B} \circ R.$$

Outcome realization needs a different map. Let $\mathsf{PhysOut}$ denote a later functor assigning physical measurement outcomes in a realized model. One would need

$$r_c : \mathcal{O}(c) \longrightarrow \mathsf{PhysOut}(R(c)),$$

natural in the appropriately declared sense. The map r is not ι. Algebra comparison and outcome interpretation have different domains and codomains.

Assumption `P3-A09` records all these bridges. None is validated in the present repository. In particular, an arbitrary label map need not preserve distinguishability, coarse graining, effects, calibration, or statistical predictions. An adequacy theorem must say what is preserved.

9.2 Why AQFT comparators come later

Haag-Kastler assigns observable algebras to already declared spacetime regions and imposes relations such as isotony and locality [13]. The original source is used here for historical provenance. The present audit did not pin a theorem-level locator in its full text.

BFV gives an exact modern formulation. Its category Man contains four-dimensional, oriented, time-oriented, globally hyperbolic Lorentzian spacetimes. Its morphisms are causal-convex, orientation-preserving, time-orientation-preserving isometric embeddings. A locally covariant theory is a functor

$$\mathcal{B} : \mathsf{Man} \longrightarrow \mathsf{Alg}$$

to unital C-star algebras with faithful unital star-homomorphisms [4, Definition 2.1]. Proposition 2.2 recovers a fixed-spacetime net.

These are rigorous targets for a future realization. They cannot serve as pre-geometric inputs because their source category already contains the spacetime structure that Paper 4 is intended to reconstruct. This is counterexample `CE-P3-03`.

9.3 Status of physical measurement

The paper has constructed no R, ι, or r for the selected physical contexts. It has proved no adequacy theorem. It has supplied no measurement calibration or operational probability. Thus P3-C05 remains **OPEN**.

The phrase "ordinary measurement localized in a realized spacetime" should be read as a target interface, not as something already available. Its use in the claim register is prospective.

10 Counterexamples as interface tests

10.1 Branch failure

Counterexample CE-P3-01 rejects the statement that every monoidal category reconstructs a unique compact group. Remove symmetry, conjugates, subobjects, direct sums, or scalar unit endomorphisms and the DR theorem is unavailable. Move to a super-Tannakian source type and the output may instead be $\mathsf{Rep}(G, \epsilon)$ for an affine supergroup scheme with parity.

Counterexample CE-P3-02 rejects canonical neutralization. The neutral affine group scheme depends on the fibre functor. Without a chosen neutralization the intrinsic object is a gerbe. Different fibre functors are related by torsors, not by a canonical physical frame.

10.2 Observable failure

Counterexample CE-P3-04 is the trivial-action witness from theorem 4.1. The same compact group acts trivially on every unital C-star algebra. It can also act nontrivially in inequivalent ways. Therefore neither observable algebra nor observable action is a field of the reconstructed group.

10.3 State and conservation failures

Counterexample CE-P3-05 is the F_2 translation action on $\ell^\infty(F_2)$. An invariant state would be an invariant mean, which does not exist. This directly forces the **OBSTRUCTED**status of the invariant-state portion of P3-C03.

Counterexample CE-P3-06 chooses independent actions α and τ. An element fixed by α need not be fixed by τ. One can also specify a symmetry action without specifying dynamics at all. Thus "invariant" and "conserved" are not interchangeable predicates.

10.4 Outcome and frame failures

Counterexample CE-P3-07 observes that a monoidal category may be given without any functor to FinSet. Monoidality does not manufacture outcomes.

Counterexample CE-P3-09 separates three types:

$$o \in \mathcal{O}(c), \qquad e \in \mathrm{Eff}(c), \qquad \omega(e) \in \mathbb{C}.$$

A label, an effect, and a scalar evaluation are not synonyms. Operational probability requires still another interpretation.

Counterexample CE-P3-08 separates fibre-functor torsors from physical reference frames. An algebraic trivialization has no built-in Hilbert space, density operator, POVM, calibration, or physical transformation rule.

ID	Blocked inference	Required repair
`CE-P3-01`	Arbitrary monoidal category to compact group	Verify the exact DR hypotheses
`CE-P3-02`	Category alone to canonical group scheme	Supply a fibre functor or use the gerbe
`CE-P3-03`	AQFT as pre-geometric reconstruction	Declare a later spacetime realization
`CE-P3-04`	Group to observable algebra	Construct $\mathcal{A}$ and α separately
`CE-P3-05`	Arbitrary action to invariant state	Add compactness, amenability, or a fixed state
`CE-P3-06`	Symmetry invariance to conservation	Declare τ, its continuity, and generator domain
`CE-P3-07`	Monoidality to outcome labels	Construct $\mathcal{O}$ and its coherence maps
`CE-P3-08`	Fibre functor to physical frame	Supply an interpretation and adequacy theorem
`CE-P3-09`	Label to effect to probability	Keep the interfaces separate

Table 4: Counterexamples are used as type checks on proposed bridges.

11 Executable witnesses

11.1 Lean scope

The file

`FoundationalReformulation/Representation.lean`

contains finite kernel-checkable interfaces for:

1. finite group actions represented by permutations;
2. invariant observables;
3. equivariant maps;
4. finite normalized states with nonnegative rational weights;
5. finite averaging;
6. invariance of the averaged state;
7. probability-free outcome-label composition;
8. the registered obstruction status of `P3-C03`.

The registry contains thirty-four claims. Exactly `P3-C03` has status **OBSTRUCTED**. Every other registered claim remains open. The Lean registry test checks this exact distribution and the single counterexample-register evidence path attached to `P3-C03`; the Markdown claim row separately names counterexample IDs `CE-P3-05` and `CE-P3-06`.

The Lean development contains no use of `sorry`, `admit`, `axiom`, `opaque`, `classical`, or `noncomputable`. Its finite proof does not formalize Haar measure or C-star algebras. It checks the permutation-reindexing core of finite averaging.

11.2 Haskell scope

The file

`src/Foundational/Representation.hs`

implements finite cyclic actions, normalized finite states, state averaging, finite observables, invariant-observable checks, finite maps, equivariance checks, and phantom-typed outcome labels. The companion tests:

1. explicitly require `validCyclicAction action` in the generator property;
2. test valid and invalid action tables;
3. test malformed weights and normalization;
4. reject NaN and positive or negative infinity;
5. admit IEEE negative zero where ordinary nonnegativity admits it;
6. probe the documented tolerance on both sides of its boundary;
7. check averaged-state invariance;
8. check invariant and noninvariant observables;
9. check equivariant maps;
10. check probability-free outcome composition and unit laws.

Haskell results are labeled **COMPUTATIONAL**. Floating-point tolerances are implementation contracts, not substitutes for the exact rational Lean theorem. Random testing explores examples; it does not prove a universal theorem.

11.3 Evidence matrix

Artifact	Status	Literal scope
Imported DR theorem	**CONDITIONAL**	Exact compact-group reconstruction if `P3-A01` and `P3-A03` hold
Lean finite averaging	**PROVED**	Finite permutation actions and normalized rational weights
Lean outcome composition	**PROVED**	Typed concatenation with no evaluation field
Haskell action/state tests	**COMPUTATIONAL**	Finite arrays, documented floating tolerance, and malformed-input rejection
Observable functor bridge	**OPEN**	`P3-C02` under `P3-A04` and `P3-A05`
Invariant state plus conservation correspondence	**OBSTRUCTED**	Exact bundled wording of `P3-C03`
Physical outcome adequacy	**OPEN**	`P3-C05` under `P3-A09`

Table 5: Imported mathematics, project proofs, computations, and open bridges are kept in different rows.

12 Prior art and exact deltas

12.1 Representation reconstruction

Tannaka's original theorem reconstructs a bicompact group from its full structured representation data [16]. The exact source locators are the dual-semigroup operations in Section 1, the first Satz 1 on printed page 2, Satz 2 on printed pages 9–10, and Satz 3 on printed pages 10–11. It does not state a theorem about an arbitrary monoidal category.

Deligne and Milne reconstruct an affine group scheme relative to an exact faithful tensor fibre functor, classify fibre functors by torsors, and describe the nonneutral gerbe [9]. Doplicher and Roberts reconstruct a compact group from a strict symmetric monoidal C-star category with the exact closure and conjugacy hypotheses [10]. Deligne's super theorem reconstructs affine supergroup-scheme data under a different characteristic-zero tensor-categorical hypothesis [8].

The project delta is not a new proof of any of these results. It is a typed placement of the DR theorem inside the proposed paper sequence, with explicit interfaces for every later physical bridge.

12.2 Observable and AQFT structures

The Doplicher-Roberts endomorphism and field-algebra results support observable-to-gauge reconstruction in a rich AQFT setting [11, 12]. Haag-Kastler and BFV organize observables after spacetime regions or spacetimes have been supplied [4, 13].

The Paper 3 delta is to avoid reversing those implications without a theorem. The observable functor and its action are declared as separate open interfaces. AQFT appears only after an explicit realization functor.

12.3 States and dynamics

Compact averaging is standard, while compact ergodic actions supply a stronger uniqueness-and-trace comparator [14]. Amenability and compact-convex fixed-point theory identify why invariant states need hypotheses [6, 7, 17]. Stone's theorem governs strongly continuous unitary dynamics on Hilbert space [15]. The weak AQFT Noether theorem of [5] has substantial spacetime and representation assumptions.

The project delta is the explicit split among:

$$\text{symmetry action}, \quad \text{state averaging}, \quad \text{natural state family},$$
$$\text{dynamics}, \quad \text{generator}, \quad \text{Noether current}.$$

Only the finite averaging core is newly kernel checked.

12.4 Outcomes and frames

Categorical quantum and operational frameworks show how much additional structure measurement semantics normally uses [1, 2]. Quantum reference-frame work begins with an explicitly Hilbert-space and probabilistic theory [3].

The Paper 3 outcome layer is weaker by construction. It retains typed finite labels and strong monoidal composition only. The frame layer retains torsor comparison data only. These are boundary interfaces, not rival reconstructions of full quantum measurement.

13 Limitations and open obligations

13.1 No constructed physical DR category

The repository does not yet construct $\mathcal{C}_{\mathrm{rep}}$ from the selected physical contexts. It does not prove symmetry, C-star enrichment, conjugates, subobjects, direct sums, strictness, or $\mathrm{End}(\mathbf{1}) = \mathbb{C}$ for such a category. Thus `P3-A01` and `P3-A03` remain **UNVALIDATED**.

No neutral fibre functor is constructed either. Assumption `P3-A02` remains **UNVALIDATED**and applies only to the neutral comparator.

13.2 No observable reconstruction

No functor $\mathcal{A}$ has been constructed from Papers 1 and 2. No natural continuous action α has been obtained from the DR equivalence. The reverse-direction AQFT sources do not provide that map. Thus `P3-A04`, `P3-A05`, and claim `P3-C02` remain open.

13.3 No preferred state or natural state family

Compact averaging transforms an exhibited state into an invariant state at one algebra. It does not choose the state. It does not make state choices natural across contexts. For noncompact nonamenable actions, invariant states may fail to exist. Thus `P3-A06` remains unvalidated and the relevant portion of `P3-C03` remains obstructed.

13.4 No dynamics from symmetry

No one-parameter evolution is produced by the compact group reconstruction. No generator is defined without a continuity hypothesis and domain. No current is produced by a fixed-point statement. Thus `P3-A07` remains unvalidated and the conservation portion of `P3-C03` remains obstructed.

13.5 No operational measurement

No physical outcome functor has been constructed for Ctx. No effects, tests, evaluator, normalization law, or probabilities are defined by the outcome-label interface. No later spacetime realization or adequacy map has been built. Thus `P3-A08`, `P3-A09`, `P3-C04`, and `P3-C05` remain open.

13.6 No semantic leap from executable models

The finite Lean and Haskell models witness that the interfaces are coherent on small finite examples. They do not show that the physical context category has finite state spaces or cyclic symmetry. They do not prove C-star positivity, norm continuity, compact Haar integration, or field-theoretic locality. They are exact where labeled and silent elsewhere.

14 Dependency map for later papers

The safe forward dependency is:

$$\begin{tikzcd}
\mathcal{C}_{\mathrm{rep}} \arrow[r, dashed, "\mathrm{DR}"] & G \\
\mathsf{Ctx} \arrow[r, dashed, "\mathcal{A}"] \arrow[d, dashed, "R"'] & \mathsf{uCStar} \arrow[u, dashed, "\alpha"'] \arrow[d, dashed, "\iota"] \\
\mathsf{Loc} \arrow[r, dashed, "\mathcal{B}"] & \mathsf{uCStar}
\end{tikzcd}$$

The upper horizontal arrow is an imported theorem only after its hypotheses are verified. The other dashed arrows are declared or future bridges. The diagram does not assert that they exist.

The outcome dependency is separate:

$$\begin{tikzcd}
\mathsf{Ctx} \arrow[r, dashed, "\mathcal{O}\ (\texttt{P3-A08})"] \arrow[d, dashed, "R"'] & \mathsf{FinSet} \arrow[d, dashed, "r"] \\
\mathsf{Loc} \arrow[r, dashed, "\mathsf{PhysOut}"] & \mathsf{FinSet}.
\end{tikzcd}$$

No arrow from $\mathcal{O}$ to Eff, no evaluation into $[0, 1]$, and no Born rule is present. Those belong to a later measurement theory.

15 Conclusion

Representation theory supplies a precise compact-group reconstruction on the Doplicher-Roberts branch. The theorem's strength comes from its exact source type: strict symmetric monoidal C-star structure, conjugates, subobjects, direct sums, and complex scalar unit endomorphisms. It does not need a fibre functor and does not output a group scheme or supergroup.

Neutral, nonneutral, and super-Tannakian reconstruction remain valuable comparators. Their input and output types stay separate. The neutral branch explains the role of a fibre functor. The nonneutral branch explains torsors and gerbes. The super branch explains why symmetry plus characteristic-zero finiteness can lead to parity-bearing affine supergroup data rather than a compact group.

The physical bridges do not come for free. An observable functor is separate data. Its natural continuous group action is separate data. An invariant state can be obtained by compact averaging only from an exhibited state, and objectwise averages need not form a natural family. Arbitrary actions can lack invariant states. Conservation belongs to an independent dynamics and generator domain. Outcome labels require a declared monoidal functor and contain no probability. Algebraic frame torsors require a later interpretation before they become physical frames. AQFT comparators begin after spacetime has already been supplied.

The result is therefore a sharper research program, not a completed emergence theorem. Claims `P3-C01`, `P3-C02`, `P3-C04`, and `P3-C05` remain **OPEN**. Claim `P3-C03` is **OBSTRUCTED**in its exact bundled form. All nine assumptions remain **UNVALIDATED**. The finite averaging and label-composition interfaces are exact executable witnesses at their stated scope, and no more.

A Source-access and locator record

A.1 Reconstruction sources

Tannaka 1939. The official J-STAGE article was checked at Section 1 for the structured dual-semigroup operations, the first Satz 1 on printed page 2, Satz 2 on printed pages 9–10, and Satz 3 on printed pages 10–11. The supported object is a bicompact topological group reconstructed from its full structured representation data.

Deligne and Milne 1982. The checked file is the corrected November 2018 author-hosted PDF retaining the original numbering. The relevant locators are Proposition 2.8 and Remark 2.10 on page 20, Theorem 2.11 on page 21, the proof conclusion on page 24, Definition 2.19 on page 25, Theorem 3.2 on pages 30–31, and Definition 3.7 and Theorem 3.9 on pages 32–33.

Doplicher and Roberts 1989 compact duality. The official Springer summary was checked. It states the strict symmetric monoidal C-star category hypotheses and the compact-group $H(G)$ conclusion. The full text was not available in the audit at theorem-level granularity. No theorem number or invented locator is supplied.

Deligne 2002. The official IAS copy was checked at Definition 0.1 on printed page 2, Proposition 0.5 and Theorem 0.6 on printed page 3, and Corollary 0.8 on printed page 4. The conclusion is affine supergroup-scheme data with parity.

A.2 Observable and spacetime comparators

Doplicher and Roberts 1989 and 1990. Official abstracts support the observable/endomorphism and field-algebra direction at a summary level. The source audit did not establish a detailed theorem locator. They are used only as reverse-direction comparators.

Haag and Kastler 1964. The official bibliographic record supplies historical provenance. The full original text was not pinned at theorem-level pages. No detailed theorem is attributed to the uninspected page range.

BFV 2003. The checked source is arXiv version 1. Section 2.2 and Definition 2.1 on printed pages 6–8 define Man, Alg, and a locally covariant theory. Proposition 2.2 on printed pages 10–12 recovers a fixed-spacetime net. The state definition and no-natural-state discussion occur on pages 15–16. Definition 3.1 is on pages 17–18. Relative Cauchy evolution and the added hypotheses preceding Theorem 4.2 are on pages 22–24, and Theorem 4.2 is on pages 24–25.

A.3 State, dynamics, outcome, and frame sources

Amenability. The classical sources are von Neumann's 1929 invariant-mean work and Day's 1957 amenable-semigroup and 1961 compact-convex fixed-point papers. They are used to identify hypotheses and the nonamenable boundary, not to claim an unqualified invariant-state theorem.

Dynamics and Noether. Stone's 1932 theorem is cited for strongly continuous unitary groups on Hilbert space. Buchholz, Doplicher, and Longo were checked at printed pages 1–2 for assumptions, page 4 for the local-implementer warning, Section 3 for local generators, and page 12 for the current-reconstruction limitation.

Outcomes. Abramsky and Heunen were checked at Section 3 on printed pages 3–5, Equation (2) on page 5, Sections 3.3 and 3.5 on pages 6–7, Section 5.3 on page 13, and Proposition 3 on page 14. Their work supports the separation of outcome and evaluation interfaces.

Reference frames. Bartlett, Rudolph, and Spekkens were checked at Section II.A on printed page 4 and Equations (2.14)–(2.15), with the conservation warning, on printed page 6. The source already assumes Hilbert and probabilistic structure.

B Audit checklist

1. The main branch is exactly Doplicher-Roberts compact-group C-star duality.
2. The DR input hypotheses and compact-group output are explicit.
3. No fibre functor is required by the DR statement.
4. Neutral, nonneutral, and super-Tannakian branches have distinct rows.
5. No affine group scheme or supergroup is attributed to DR.
6. The observable functor is separately declared.
7. The group action is separately declared and naturality is written.
8. Every averaged state is exhibited before averaging.
9. Compact averaging does not become a preferred natural family.
10. The nonamenable invariant-state obstruction is recorded.
11. Symmetry and one-parameter dynamics use different symbols and types.

12. The generator domain is written.
13. Fixed-point equivalence is not called Noether's theorem.
14. Outcome labels are probability free.
15. Effects and evaluations are separate future interfaces.
16. Fibre-functor torsors are called algebraic frame data.
17. Physical frames require later interpretation.
18. Spacetime realization maps are separately typed.
19. Haag-Kastler and BFV are later comparators.
20. All registered statuses match the ledgers.

References

[1] Samson Abramsky and Bob Coecke. A categorical semantics of quantum protocols. In *Proceedings of the 19th Annual IEEE Symposium on Logic in Computer Science*, pages 415–425, 2004. doi: 10.1109/LICS.2004.1319636. URL https://arxiv.org/abs/quant-ph/0402130v2.

[2] Samson Abramsky and Chris Heunen. Operational theories and categorical quantum mechanics. In *Categorical Quantum Models and Logics*, pages 49–101. Cambridge University Press, 2019. doi: 10.1017/CBO9781139519687.007. URL https://arxiv.org/abs/1206.0921v3.

[3] Stephen D. Bartlett, Terry Rudolph, and Robert W. Spekkens. Reference frames, superselection rules, and quantum information. *Reviews of Modern Physics*, 79(2):555–609, 2007. doi: 10.1103/RevModPhys.79.555. URL https://arxiv.org/abs/quant-ph/0610030v3.

[4] Romeo Brunetti, Klaus Fredenhagen, and Rainer Verch. The generally covariant locality principle: A new paradigm for local quantum physics. *Communications in Mathematical Physics*, 237:31–68, 2003. doi: 10.1007/s00220-003-0815-7. URL https://arxiv.org/abs/math-ph/0112041v1.

[5] Detlev Buchholz, Sergio Doplicher, and Roberto Longo. On Noether's theorem in quantum field theory. *Annals of Physics*, 170(1):1–17, 1986. doi: 10.1016/0003-4916(86)90086-2. URL https://www-library.desy.de/preparch/desy/postpr/1985/desy85-023.pdf.

[6] Mahlon M. Day. Amenable semigroups. *Illinois Journal of Mathematics*, 1(4):509–544, 1957. doi: 10.1215/ijm/1255380675. URL https://doi.org/10.1215/ijm/1255380675.

[7] Mahlon M. Day. Fixed point theorems for compact convex sets. *Illinois Journal of Mathematics*, 5(4):585–590, 1961. doi: 10.1215/ijm/1255631582. URL https://doi.org/10.1215/ijm/1255631582.

[8] Pierre Deligne. Catégories tensorielles. *Moscow Mathematical Journal*, 2(2):227–248, 2002. URL https://publications.ias.edu/book/export/html/434.

[9] Pierre Deligne and James S. Milne. Tannakian categories. In *Hodge Cycles, Motives, and Shimura Varieties*, volume 900 of *Lecture Notes in Mathematics*, pages 101–228. Springer, 1982. URL https://www.jmilne.org/math/xnotes/tc2018.pdf. Corrected 2018 version retaining the original numbering.

[10] Sergio Doplicher and John E. Roberts. A new duality theory for compact groups. *Inventiones Mathematicae*, 98:157–218, 1989. doi: 10.1007/BF01388849. URL https://doi.org/10.1007/BF01388849.

[11] Sergio Doplicher and John E. Roberts. Endomorphisms of C*-algebras, cross products, and duality for compact groups. *Annals of Mathematics*, 130(1):75–119, 1989. doi: 10.2307/1971477. URL https://doi.org/10.2307/1971477.

[12] Sergio Doplicher and John E. Roberts. Why there is a field algebra with a compact gauge group describing the superselection structure in particle physics. *Communications in Mathematical Physics*, 131:51–107, 1990. doi: 10.1007/BF02097680. URL https://doi.org/10.1007/BF02097680.

[13] Rudolf Haag and Daniel Kastler. An algebraic approach to quantum field theory. *Journal of Mathematical Physics*, 5(7):848–861, 1964. doi: 10.1063/1.1704187. URL https://doi.org/10.1063/1.1704187.

[14] Raphael Høegh-Krohn, Magnus Landstad, and Erling Størmer. Compact ergodic groups of automorphisms. *Annals of Mathematics*, 114(1):75–86, 1981. doi: 10.2307/1971377. URL https://doi.org/10.2307/1971377.

[15] Marshall H. Stone. On one-parameter unitary groups in hilbert space. *Annals of Mathematics*, 33(3): 643–648, 1932. doi: 10.2307/1968538. URL https://doi.org/10.2307/1968538.

[16] Tadao Tannaka. Über den dualitätssatz der nichtkommutativen topologischen gruppen. *Tohoku Mathematical Journal*, 45:1–12, 1939. URL https://www.jstage.jst.go.jp/article/tmj1911/45/0/45_0_1/_article.

[17] John von Neumann. Zur allgemeinen theorie des masses. *Fundamenta Mathematicae*, 13:73–116, 1929. doi: 10.4064/fm-13-1-73-116. URL https://doi.org/10.4064/fm-13-1-73-116.