Quantum Mechanics from Categorical Representations and Compositional Measurement

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23 Jul 2026

Abstract

Paper 3 supplies categorical representation data, probability-free outcome labels, symmetry actions, and optional exhibited algebraic states. It does not supply an operational probability theory. This paper makes that missing step explicit through an unvalidated *operational bridge*. The bridge carries systems, tests and outcomes, transformations, serial and parallel composition, operational equivalence, states, effects, closed-circuit evaluation, convex mixing, discarding, and finite-dimensional ordered vector spaces. It also contains a typed interpretation of selected Paper 3 labels as effects. None of these fields is inferred from a monoidal category.

The main reconstruction route is the finite-dimensional operational-probabilistic theorem of Chiribella, D'Ariano, and Perinotti. Its six principles are kept separate: causality, perfect distinguishability, ideal compression, local distinguishability, pure conditioning, and purification with essential uniqueness. The literature theorem reconstructs complex density matrices, effects, and completely positive trace-nonincreasing transformations under those exact hypotheses. The repository has not constructed a bridge satisfying them, so the corresponding project claims remain open.

Barnum and Wilce supply a separate comparison route. It requires finite-dimensional state-complete homogeneous self-dual models, factorizable self-duality or the full dagger-HSD alternative, suitable non-signalling locally tomographic composites, and an actual qubit. Its complex conclusion retains direct-sum superselection. Gleason is used only after Hilbert structure exists and only in dimension at least three. Qubits use the stronger all-effects and POVM-noncontextual hypotheses of Busch and Caves, Fuchs, Manne, and Renes.

Real, complex, quaternionic, dimension-two, infinite-dimensional, and superselection cases are displayed in a route table rather than collapsed into one theorem. That singular master-classification claim is **OBSTRUCTED**. Reversible channels are also separated from a chosen time evolution, and POVMs are separated from instruments

and interpretations. The accompanying Lean development proves only finite rational normalization, channel composition, product normalization, coarse-graining, and effect bounds. Its record named `OperationalBridge` is a non-enforcing finite schema projection, not a formalization of the mathematical bridge below. Haskell provides computational tests and counterexample witnesses. Neither executable layer formalizes an imported reconstruction theorem.

Contents

1 Question and answer

1.1 Research question

The question is:

> Which additional compositional and probabilistic assumptions turn the probability-free representation and outcome interfaces of Paper 3 into finite-dimensional complex quantum mechanics?

There is a sharp positive theorem in the literature once a complete operational-probabilistic framework and six information-theoretic principles are supplied. No theorem in the audited source set constructs those data from the earlier categorical interfaces. The bridge is therefore an assumption package, not a change of notation.

This paper treats reconstruction as a typed dependency question. The expression "from categorical representations" names the source of the bridge. It does not mean that category theory alone forces complex Hilbert space, probabilities, tensor products, unitary time evolution, or a measurement interpretation.

1.2 Registered claims

1.3 Status discipline

The imported theorems below are proved results in their source literature. Their application to the proposed physical contexts is conditional on assumptions that remain **UNVALIDATED**. The project claims therefore stay **OPEN**unless an obstruction directly defeats their exact wording. The exception is `P5-C05`. The audited theorems have incompatible domains, and one of them retains superselection rather than excluding it. The requested single classification theorem is therefore **OBSTRUCTED**.

Claim	Registered target	Status
`P5-C01`	An explicitly supplied strong-monoidal operational bridge connects the Paper 3 observable/outcome interface to a finite-dimensional operational-probabilistic theory with well-defined composition.	**OPEN**
`P5-C02`	The finite-dimensional operational bridge plus the six separate CDP principles reconstruct complex density-matrix state spaces, effects, and completely positive trace-nonincreasing transformations.	**OPEN**
`P5-C03`	In the finite state-complete HSD comparison route, factorizable or dagger-HSD composites, local tomography, and a qubit select complex finite-dimensional C-star/Jordan systems while retaining direct-sum superselection.	**OPEN**
`P5-C04`	Downstream of Hilbert/effect structure, noncontextual additive probability measures have Born trace form: projection measures in dimension at least three and all-effects/POVM valuations for dimension two.	**OPEN**
`P5-C05`	One reconstruction theorem classifies or excludes real, quaternionic, low-dimensional, infinite-dimensional, and superselection alternatives.	**OBSTRUCTED**
`P5-C06`	The compositional reconstruction does not by itself select a measurement interpretation or solve the measurement problem.	**OPEN**

Table 1: Paper 5 claim status. Literature results do not validate project assumptions. The obstruction evidence for `P5-C05` is the counterexample register.

The finite Lean lemmas are **PROVED**at their exact rational, finite-carrier scope. The Haskell tests are **COMPUTATIONAL**. Neither status transfers to CDP, Gleason, Piron, Soler, Hanche-Olsen, or Barnum-Wilce.

2 The inherited interface is probability free

2.1 Inputs from Papers 1 through 3

Papers 1 and 2 provide proposed context, composition, coverage, descent, and condensed-realization interfaces. Their registered physical claims remain open. Paper 3 adds a candidate representation category, a conditional compact symmetry reconstruction, a separately declared observable-algebra functor

$$\mathcal{A} : \mathsf{Ctx} \longrightarrow \mathsf{CStar},$$

a separately declared action α, optional exhibited algebraic states, and an outcome-label functor

$$\mathcal{O} : \mathsf{Ctx} \longrightarrow \mathsf{FinSet}.$$

The observable functor, action, and outcome functor are themselves open project bridges.

The outcome type is intentionally weak. For each context c, $\mathcal{O}(c)$ names possible labels and supports composition maps

$$\mathcal{O}(c) \times \mathcal{O}(d) \longrightarrow \mathcal{O}(c \otimes d).$$

It contains no ordered scalar object, effect cone, evaluation pairing, normalization law, or operational equivalence.

Proposition 2.1 (Inherited signature boundary)**.** *The Paper 3 outcome-label signature does not project a probability evaluation.*

Proof. Its fields are label types, finite-word constructors, and composition and unit maps. No field has codomain $[0, 1]$. No field maps a label to an effect. No relation identifies preparations or measurement events by all closed-circuit probabilities. The desired evaluation is absent from the signature. □

This proposition is a signature observation, not a physical impossibility theorem. An interpretation map may be added. It must be added and tested.

2.2 What cannot be imported

None of the following follows from the inherited interface:

- a convex state space or an ordered effect space;
- a state-effect pairing in $[0, 1]$;
- tests whose outcome probabilities sum to one;
- operational equivalence of preparations, effects, or transformations;

- instruments or conditional state-update maps;
- positivity or complete positivity;
- causal discarding, no-signalling, or local tomography;
- purification or compression;
- an inner product, dagger, or biproduct;
- a real, complex, or quaternionic scalar choice;
- Hilbert space, a Born rule, or a Hamiltonian;
- a measurement ontology.

An algebraic state on a C-star algebra does not repair the gap by itself. It supplies a positive normalized functional after a C-star algebra has already been chosen. An operational theory also needs tests, transformations, composites, equivalence, and a statement connecting evaluations to observed frequencies or probabilities.

3 The operational bridge

3.1 Data

Definition 3.1 (Operational bridge)**.** Let $(\mathsf{Ctx}, \otimes, I)$ be the context category and let $\mathcal{O} : \mathsf{Ctx} \to \mathsf{FinSet}$ be the Paper 3 outcome-label functor. An *operational bridge* consists of the following supplied data.

1. A symmetric monoidal system category $(\mathsf{Sys}, \boxtimes, \mathbf{1})$ and a strong symmetric monoidal map
$$J : \mathsf{Ctx} \longrightarrow \mathsf{Sys}.$$
2. For systems A, B, transformation and test types
$$\mathsf{Transf}(A, B), \qquad \mathsf{Tests}(A, B),$$
with every test carrying a finite set of classical outcomes and one transformation for each outcome.
3. Operational equivalence relations on preparations, effects, and transformations, defined by equality of all compatible closed-circuit probabilities.
4. Quotient state and effect spaces
$$\mathsf{St}(A), \qquad \mathsf{Eff}(A),$$
together with an evaluation
$$(\,\cdot\,|\,\cdot\,)_A : \mathsf{Eff}(A) \times \mathsf{St}(A) \longrightarrow [0, 1].$$

5. Serial composition $\circ$, parallel composition $\boxtimes$, and their units.
6. Coarse-graining of selected test outcomes and convex randomization of preparations, tests, and transformations.
7. A deterministic effect $u_A \in \mathsf{Eff}(A)$ for every system, with monoidal discarding data.
8. A closed-circuit evaluator
$$\mathrm{close} : \mathsf{Transf}(\mathbf{1}, \mathbf{1}) \longrightarrow [0, 1].$$
9. Finite-dimensional ordered real vector spaces generated by the state and effect quotients.
10. A typed interpretation
$$\iota_c : \mathcal{O}(c) \longrightarrow \mathsf{Eff}(Jc)$$
for the selected Paper 3 labels, together with declared test partitions and monoidal compatibility.

The mathematical definition above is a target interface. The Lean record named `OperationalBridge` does not mirror or formalize that complete list. It is a non-enforcing finite schema projection with carrier types, raw object-level tensor operations, selected operations, a list-valued test-to-effects accessor, and a label-to-effect map. It contains no context or system category, no action on morphisms, no symmetric monoidal coherence, no per-outcome transformation accessor for a test, and no preparation or effect equivalence relation. Its fields ending in `: Prop` are unparameterized proposition slots chosen by the caller. They are not proofs and do not state equations involving the preceding operations. The record therefore checks neither the target interface nor any operational law. Its limited purpose is to keep a finite vocabulary visible beside the kernel-checked finite algebra.

3.2 Laws

The bridge laws include:
$$(h \circ g) \circ f = h \circ (g \circ f), \tag{1}$$
$$(f_2 \boxtimes g_2) \circ (f_1 \boxtimes g_1) = (f_2 \circ f_1) \boxtimes (g_2 \circ g_1), \tag{2}$$
$$\sum_{x \in X} (e_x|\rho) = 1 \quad \text{for every normalized test } X, \tag{3}$$
$$\left(\sum_{x \in C} e_x \middle| \rho \right) = \sum_{x \in C} (e_x|\rho), \tag{4}$$
$$u_{A \boxtimes B} = u_A \boxtimes u_B. \tag{5}$$
Operational equivalence must be a congruence for both compositions and for coarse-graining. Convex mixing must agree with evaluation:
$$(e|p\rho + (1-p)\sigma) = p(e|\rho) + (1-p)(e|\sigma), \qquad 0 \le p \le 1.$$

These are not decorative coherence equations. Without them, quotient states and effects may depend on representatives, a coarse event may fail additivity, and composite circuits may have ambiguous probabilities. They belong to the paper-level target interface. The similarly named Lean proposition slots neither encode these formulas nor verify them.

3.3 Strong monoidality does not imply tomography

The strong monoidal map J identifies how context composition is represented as system composition. It does not say that local effects separate bipartite states. That property is local distinguishability in the CDP route and local tomography in the Barnum-Wilce route. Both are separate assumptions.

Likewise, the map ι_c need not be surjective. Selected Paper 3 labels may designate only a small family of effects. If they are claimed to generate or tomographically determine the full effect space, a further assumption is needed.

3.4 Bridge status

Assumptions `P5-A01`, `P5-A02`, and `P5-A03` state the signature, laws, and finite ordered quotient. All three are **UNVALIDATED**. The repository contains no physical model of Papers 1 through 3 on which this bridge has been constructed. Claim `P5-C01` therefore remains **OPEN**.

4 Finite operational algebra

The kernel-checked layer records algebraic consequences that do not require a quantum reconstruction. Let X be a finite type.

Definition 4.1 (Finite distribution and effect)**.** A finite distribution is a function

$$p : X \longrightarrow \mathbb{Q}_{\geq 0}, \qquad \sum_{x \in X} p(x) = 1.$$

A finite effect is a function

$$e : X \longrightarrow \mathbb{Q}, \qquad 0 \leq e(x) \leq 1.$$

Its evaluation is

$$\langle e, p \rangle = \sum_{x \in X} p(x) e(x).$$

Proposition 4.2 (Finite effect bounds)**.** *For every finite distribution p and effect e,*

$$0 \leq \langle e, p \rangle \leq 1.$$

Proof. Every summand is nonnegative. The pointwise bound $e(x) \leq 1$ gives

$$\sum_x p(x) e(x) \leq \sum_x p(x) = 1.$$

Lean checks the lower and upper bounds in two theorems:

`evaluateEffect_nonnegative`
`evaluateEffect_atMostOne`.

□

Definition 4.3 (Finite channel)**.** For finite X, Y, a channel in row-stochastic convention is a function

$$K : X \times Y \longrightarrow \mathbb{Q}_{\geq 0}$$

such that

$$\sum_{y \in Y} K(x, y) = 1$$

for each x. Its action on a distribution is

$$(K_* p)(y) = \sum_{x \in X} p(x) K(x, y).$$

Proposition 4.4 (Channel normalization)**.** *If p is normalized and K is row stochastic, then $K_* p$ is normalized.*

Proof. Finite sums may be exchanged:

$$\begin{aligned} \sum_y (K_* p)(y) &= \sum_y \sum_x p(x) K(x, y) \\ &= \sum_x p(x) \sum_y K(x, y) \\ &= \sum_x p(x) = 1. \end{aligned}$$

Nonnegativity follows termwise. The Lean constructor checks the proof. Its name is `FiniteChannel.apply`. ☐

Proposition 4.5 (Sequential composition)**.** *For channels $K : X \to Y$ and $L : Y \to Z$, define*

$$(L \circ K)(x, z) = \sum_{y \in Y} K(x, y) L(y, z).$$

Then

$$(L \circ K)_* p = L_* (K_* p),$$

and the identity channel acts as the identity on distributions.

Proof. Both statements follow by finite sum exchange and distributivity. Lean checks them as `FiniteChannel.comp_apply` and `FiniteChannel.identity_apply`. ☐

Proposition 4.6 (Parallel product)**.** *For finite distributions p on X and q on Y, the function*

$$(p \otimes q)(x, y) = p(x) q(y)$$

is a normalized distribution on $X \times Y$.

Proof. Nonnegativity is immediate and

$$\sum_{x,y} p(x) q(y) = \left(\sum_x p(x) \right) \left(\sum_y q(y) \right) = 1.$$

Lean checks the statement in `tensorDistribution`. ☐

Definition 4.7 (Finite coarse-graining)**.** For a map $f : X \to Y$, define

$$(f_* p)(y) = \sum_{x: f(x)=y} p(x).$$

This is the action of the deterministic channel

$$K_f(x, y) = \begin{cases} 1, & f(x) = y, \\ 0, & f(x) \neq y. \end{cases}$$

Normalization is inherited from theorem 4.4. Lean proves the identity law

$$(\mathrm{id}_X)_* p = p.$$

The Haskell suite tests identity and composition of explicit coarse maps.

4.1 What the finite layer does not prove

These propositions hold for classical finite probability vectors. They are useful interface checks because any operational bridge must at least respect normalization and composition on its finite classical control. They do not prove purification, ideal compression, homogeneous self-duality, complex scalar selection, or complete positivity. The finite layer is deliberately below every reconstruction theorem used later.

5 The six CDP principles

Chiribella, D'Ariano, and Perinotti work in an operational-probabilistic framework in which experiments are circuits, outcomes are classical, and closed circuits receive probabilities [8]. States and effects are operational equivalence classes. The real state and effect spaces are finite-dimensional.

5.1 Causality

`P5-A04` is:

> Preparation probabilities do not depend on a later choice of observation. Equivalently, every system has a unique deterministic effect.

The equivalence uses the source framework. It is stronger than the existence of some discard map. Uniqueness controls marginalization and temporal ordering of tests. The Paper 3 category has no such result.

5.2 Perfect distinguishability

`P5-A05` says that every state that is not completely mixed is perfectly distinguishable from another state. The terms "completely mixed" and "perfectly distinguishable" use the operational state and effect spaces. They cannot be stated on labels alone.

5.3 Ideal compression

`P5-A06` requires a lossless and maximally efficient compression for every state. Lossless means that encoding followed by decoding fixes the entire face generated by the state. Maximal efficiency says the encoding system contains no states irrelevant to that face. Compression is not a generic retraction in the context category.

5.4 Local distinguishability

`P5-A07` says that distinct bipartite states can be separated by local effects. In the finite setting this supplies the dimension relation

$$D_{AB} = D_A D_B.$$

The principle is often called local tomography. Its mathematical role is not exhausted by monoidal composition. Real quantum theory is a standard witness to the difference.

5.5 Pure conditioning

`P5-A08` states that conditioning a pure bipartite state on an atomic effect gives a pure conditional state when the result is nonzero. The premise uses a notion of purity and an atomic effect in the operational cone. The conclusion concerns an unnormalized conditional state before renormalization.

5.6 Purification with essential uniqueness

`P5-A09` has two parts. Every mixed state is the marginal of a pure bipartite state. When two purifications use the same purifying system, a reversible transformation on that system relates them. Existence without the fixed-purifier uniqueness clause is not the CDP postulate.

5.7 Independence in the ledger

The six principles have separate IDs because they can fail separately. Bundling them under "purification assumptions" would conceal the exact failure location. No principle is marked established. No finite test validates a universally quantified principle over the intended physical theory.

6 The imported finite reconstruction

Theorem 6.1 (Imported CDP reconstruction)**.** *In the finite-dimensional operational-probabilistic framework of [8], assume causality, perfect distinguishability, ideal compression, local distinguishability, pure conditioning, and purification with essential uniqueness. Then each normalized state space is the full density-matrix state space on $\mathbb{C}^{d_A}$. Effects are the usual quantum effects. Transformations $A \to B$ are completely positive trace-nonincreasing maps between the associated matrix algebras.*

The state-space conclusion is Theorem 20 of the source. The transformation conclusion is Corollary 52. The source definitions distinguish general trace-nonincreasing operations from trace-preserving channels. One sentence in the printed proof of Corollary 52 says "trace-preserving" for general operations. The corollary statement and the earlier definitions control the quotation; the proof sentence is treated as a typographical slip.

6.1 Project application

If assumptions `P5-A01` through `P5-A09` were validated for one bridge, theorem 6.1 would give a conditional complex finite reconstruction. No such validation exists. Therefore `P5-C02` remains **OPEN**.

The literature theorem itself is not re-proved in Lean. The paper cites it with its exact finite operational domain. This avoids the common mistake of treating a successful finite stochastic test as evidence for purification or complex matrix classification.

7 States, effects, operations, and instruments

The reconstructed objects form a hierarchy:

$$\begin{aligned} E &: \text{ effect, giving one outcome probability,} \\ \{E_i\}_i &: \text{ POVM, with } \textstyle\sum_i E_i = I, \\ \mathcal{I}_i &: \text{ CP trace-nonincreasing operation,} \\ \{\mathcal{I}_i\}_i &: \text{ instrument, with } \textstyle\sum_i \mathcal{I}_i \text{ trace preserving,} \\ \mathcal{C} &: \text{ channel, a CP trace-preserving map.} \end{aligned}$$

For a density matrix ρ,

$$p(i|\rho) = \mathrm{Tr}(\rho E_i).$$

An instrument also supplies an unnormalized conditional output

$$\mathcal{I}_i(\rho).$$

Its effect is fixed by

$$\mathrm{Tr}(\mathcal{I}_i(\rho)) = \mathrm{Tr}(\rho E_i),$$

but this equality does not determine $\mathcal{I}_i$.

Example 7.1 (Same probabilities, different updates)**.** Let $\{P_i\}$ be a projective measurement and let σ_i be arbitrary output density matrices. Define

$$\mathcal{I}_i(\rho) = \mathrm{Tr}(P_i \rho)\sigma_i.$$

Every choice of the σ_i has outcome probabilities $\mathrm{Tr}(P_i \rho)$. Different choices give different conditional output states. The sum is trace preserving.

This is counterexample `CE-P5-10`. The Haskell test suite implements the finite classical measure-and-prepare analogue. Two instruments share the same effect rows and outcome distribution but return different prepared states.

Davies and Lewis introduced instruments as operation-valued measures [9]. Ozawa relates completely positive instruments to measurement processes in a Hilbert-space setting [18]. These are process theorems, not assertions that a physical collapse has been selected.

8 The Born pairing in the main route

Under theorem 6.1, the operational state-effect pairing is represented as

$$(E|\rho) = \mathrm{Tr}(\rho E).$$

This is the Born probability rule in the reconstructed matrix representation. It is not obtained by reading an arbitrary categorical scalar as a probability. The order interval $0 \leq E \leq I$, positivity of ρ, trace-one normalization, and operational evaluation have all entered before the formula is available.

The formula does not choose the complex field independently of the CDP theorem. It also does not select a unique instrument for a POVM. Probability representation and conditional state update remain different questions.

9 Gleason after Hilbert structure

9.1 Projection measures in dimension at least three

Theorem 9.1 (Imported Gleason theorem)**.** *Let $\mathcal{H}$ be a separable real or complex Hilbert space with $\dim \mathcal{H} \geq 3$. Let μ be a normalized countably additive nonnegative measure on its closed subspaces, equivalently on orthogonal projections. Then there is a positive trace-class operator ρ with $\mathrm{Tr}\,\rho = 1$ such that*

$$\mu(P) = \mathrm{Tr}(\rho P).$$

This is the scope of Gleason's theorem [13, Theorems 3.5 and 4.1]. The Hilbert space and its orthogonality lattice are inputs. The theorem does not reconstruct Hilbert space, choose complex scalars, construct transformations, or cover dimension two.

9.2 The qubit requires a stronger event domain

For a qubit, projection-only frame functions admit dispersion-free assignments that defeat the standard theorem. Busch instead assumes a valuation on all effects $0 \leq E \leq I$, normalized and countably additive on effect families whose sum is at most the identity [6]. Caves, Fuchs, Manne, and Renes use a finite-dimensional frame function on the full effect set, noncontextual across every finite POVM and normalized on each POVM [7].

Theorem 9.2 (Imported all-effects trace representation)**.** *On a finite-dimensional complex Hilbert space, including dimension two, a positive normalized valuation on all effects that is additive and noncontextual across all finite POVMs has the form*

$$f(E) = \mathrm{Tr}(\rho E)$$

for a unique density operator ρ.

The qubit theorem is not standard Gleason with a lower dimension bound. It changes the measurement domain and assumes all-effects noncontextuality. Assumptions `P5-A10` and `P5-A11` keep the two routes separate. Both remain **UNVALIDATED**, so `P5-C04` remains **OPEN**.

10 The Barnum-Wilce comparison route

10.1 Homogeneous self-dual cones

Let A be a finite-dimensional order-unit space with positive cone A_+. Homogeneity says that the automorphism group of A_+ acts transitively on its interior. Self-duality says that some inner product identifies A_+ with its dual cone. The Koecher-Vinberg theorem then gives a Euclidean Jordan algebra structure.

This conclusion does not yet select complex matrices. Euclidean Jordan algebras include self-adjoint real, complex, and quaternionic matrices, spin factors, the exceptional Albert algebra, and direct sums.

10.2 Exact comparison hypotheses

The first Barnum-Wilce route uses:

1. finite-dimensional state-complete order-unit models;
2. homogeneous state cones;
3. specified self-dualizing inner products;
4. factorizability of the composite self-dualizing form;
5. suitable non-signalling composites;
6. local tomography for every relevant pair;
7. an actual qubit object $M_2(\mathbb{C})_{\mathrm{sa}}$.

Their categorical variant instead assumes the complete dagger-HSD structure used in the proposition. The two lists cannot be combined by keeping only the word "HSD."

Theorem 10.1 (Imported Barnum-Wilce comparison). *In the finite state-complete HSD framework of [3], factorizable self-duality, suitable locally tomographic composites, and a qubit imply that systems are self-adjoint parts of finite-dimensional complex C-star algebras. The stated dagger-HSD categorical variant reaches the corresponding standard complex quantum category under its own full hypotheses.*

The source locators are Proposition 1 and Proposition 16. The qubit-dependent complex-selection step is related to Hanche-Olsen's tensor-product theorem [14].

10.3 Superselection survives

A finite-dimensional complex C-star algebra has the form

$$\bigoplus_{k=1}^{r} M_{n_k}(\mathbb{C}).$$

The direct summands are superselection sectors. Barnum and Wilce explicitly retain quantum theory with superselection rules. To obtain one simple factor $M_n(\mathbb{C})$, a trivial-center or irreducibility premise is needed.

Assumptions `P5-A14` through `P5-A19` record the exact comparison route. None is validated for a Paper 3 model. Claim `P5-C03` stays **OPEN**.

11 Real and quaternionic alternatives

11.1 Real systems

For real d-dimensional quantum theory, the real dimension of the self-adjoint matrix space is

$$D_{\mathbb{R}}(d) = \frac{d(d+1)}{2}.$$

For nontrivial m, n,

$$D_{\mathbb{R}}(mn) > D_{\mathbb{R}}(m)D_{\mathbb{R}}(n).$$

Products of local effects therefore do not access all global degrees of freedom. Real quantum theory fails local tomography. Hardy and Wootters show that it is bilocally tomographic [16]. The failure is not an inconsistency.

11.2 Quaternionic systems

Quaternionic quantum mechanics is also a systematic formal alternative [2]. The noncommutativity of quaternionic scalars makes an ordinary tensor product problematic, a point already central in the foundational formulation of Finkelstein, Jauch, Schiminovich, and Speiser [11].

Barnum, Graydon, and Wilce construct larger Jordan categories containing real, complex, and quaternionic systems with nonstandard composites [4]. These composites need not be locally tomographic. Two complex systems may carry an extra classical bit, and a quaternionic pair may have a real composite.

The defensible conclusion is conditional:

> Within the declared finite HSD/Jordan class, standard locally tomographic composites with the remaining Barnum-Wilce hypotheses and a qubit select the complex branch.

The statement "quaternionic quantum mechanics is inconsistent" is false.

12 The quantum-logic comparison

12.1 Piron

Piron's representation route starts with an irreducible complete atomistic orthomodular lattice satisfying the covering law and having rank at least four. It concludes that the lattice is represented by biorthogonally closed subspaces of a generalized Hilbert space over an involutive division ring [19, 24].

Paper 3 supplies no proposition lattice, atoms, orthocomplementation, covering law, irreducibility, or rank. The route is therefore a comparator, not an alternative proof of `P5-C02`.

12.2 Soler

Soler starts with an orthomodular Hermitian space over an involutive division ring and assumes an infinite orthonormal sequence [17, 21]. The conclusion restricts the scalar field to

$$\mathbb{R}, \qquad \mathbb{C}, \qquad \mathbb{H}.$$

It does not select $\mathbb{C}$.

Infinite dimension without the orthonormal-sequence hypothesis is insufficient. Holland discusses Keller-type counterexamples over nonclassical involutive division rings. The Piron-Soler route also has no direct qubit coverage. Assumptions `P5-A20` and `P5-A21` remain **UNVALIDATED**and are not inserted into the CDP proof.

13 Case table and obstruction

Case	Audited result	Honest conclusion
Finite complex, including dimension two	CDP	Density matrices on $\mathbb{C}^d$, effects, CP trace-nonincreasing transformations, instruments, channels, and unitary reversible maps, conditional on the full bridge and six principles.
Finite complex through Jordan theory	Barnum-Wilce	Complex finite C-star/Jordan systems under the complete HSD, factorization or dagger, composite, tomography, and qubit hypotheses. Direct sums remain.

Case	Audited result	Honest conclusion
Finite real	Parameter and tomography comparison	Excluded by the selected local-tomography premises, but coherent under weaker tomography.
Finite quaternionic	Quaternionic and Jordan comparator literature	Naive composition is problematic and local tomography fails, but coherent nonstandard Jordan composites exist.
Dimension-two probability	Busch and CFMR	Trace form follows for noncontextual normalized valuations on all effects and POVMs. Projection-only Gleason does not apply.
Rank below four quantum logic	Piron limitation	Outside the representation theorem.
Infinite generalized Hilbert space	Piron plus Soler	With exact lattice, form, and infinite orthonormal-sequence hypotheses, scalars reduce only to $\mathbb{R}, \mathbb{C}, \mathbb{H}$.
Infinite operator-algebraic quantum theory	No selected theorem	Outside CDP and Barnum-Wilce. Topology, normality, sigma-additivity, domains, and C-star or W-star analysis are new work.
Finite superselection	Finite complex C-star structure	Direct sums of matrix algebras are allowed. An added trivial-center premise is needed to force a factor.
Interpretation and collapse	No reconstruction theorem	No interpretation is selected. A collapse theory needs additional dynamics or ontology.

Table 2: Route-separated scalar, dimensional, and interpretive cases.

Proposition 13.1 (Master-classification obstruction)**.** *No single theorem in the audited route set simultaneously classifies or excludes all cases in table 2.*

Proof. CDP and Barnum-Wilce are finite-dimensional. Piron has a rank lower bound and first returns a generalized Hilbert space. Soler needs an infinite orthonormal sequence and leaves three scalar fields. Standard Gleason excludes dimension two. The all-effects theorem repairs the probability representation only after Hilbert/effect structure is supplied. Barnum-Wilce retains direct-sum superselection. These conclusions arise from incompatible source domains and cannot be conjoined into one theorem application. □

This is the obstruction registered as `CE-P5-12`. It establishes the **OBSTRUCTED**status of `P5-C05`. The case table is the repair.

14 Categorical quantum mechanics is a semantics comparator

Abramsky and Coecke work with strongly compact closed categorical structure and biproducts [1]. The framework gives a compositional semantics for teleportation, entanglement swapping, measurement branching, and an abstract Born calculus. It is a semantics theorem, not a uniqueness theorem.

Compact and dagger-compact syntax has models beyond finite-dimensional complex quantum theory. Real inner-product spaces, relations, and suitable semiring-module models share relevant structure. The category of general unnormalized CP maps also differs from the deterministic trace-preserving subcategory. Cups and caps need not be trace-preserving channels.

The operational bridge may be compared with categorical quantum mechanics after reconstruction. It may not use compact closure or monoidality as a substitute for CDP's causality, tomography, purification, or scalar-selection content.

15 Reversibility is not time evolution

Within the finite complex operational theory, reversible channels are unitary conjugations:

$$\rho \longmapsto U\rho U^*.$$

This classifies allowed reversible transformations. It does not choose a one-parameter family.

Time evolution requires separate data

$$\tau : (\mathbb{R}, +) \longrightarrow \mathrm{Aut}(A)$$

or, in Hilbert form,

$$U : \mathbb{R} \longrightarrow \mathcal{U}(\mathcal{H}),$$

together with strong continuity. Stone's theorem then supplies a self-adjoint generator H such that

$$U(t) = e^{-itH}$$

up to sign convention [23]. Stone does not select H.

Assumption `P5-A22` records the chosen dynamics and its generator domain. Hardy's reconstruction also states that no particular Hamiltonian is recovered [15]. This gives counterexample `CE-P5-09`: the same Hilbert space supports many inequivalent one-parameter groups.

Arbitrary channels are not reversible dynamics. Instruments are families of outcome-conditioned operations. Unitary conjugations form only one part of the transformation space.

16 State update and dilation

P5-A23 says that a conditional update is supplied through an instrument or a measurement process. It is not inferred from an effect. Theorem 7.1 gives a finite witness.

Stinespring's theorem represents completely positive maps through a larger Hilbert space and a representation [22]. Kraus and isometric forms make such dilations concrete in finite dimension. A dilation is not unique. It explains how a channel can arise from an enlarged process, but it does not decide whether collapse is physical, epistemic, relative, or absent.

The distinction also prevents a typing error:

$$E_i \quad \text{and} \quad \mathcal{I}_i$$

have different domains and codomains. The first is an effect. The second is a state-transforming operation.

17 Interpretation boundary

The reconstructed operational signature contains density matrices, effects, probabilities, transformations, and instruments. It contains no:

- actual configuration or beable;
- preferred branch or outcome;
- primitive stochastic collapse law;
- observer-system cut;
- unique instrument attached to every POVM;
- ontology for the density operator.

Everett adds a relative-state interpretation to unitary quantum mechanics [10]. Bohm adds configuration variables and guidance dynamics [5]. Ghirardi, Rimini, and Weber add a stochastic collapse dynamics [12]. The additions are mutually different while sharing or closely matching the ordinary operational predictions in their intended domains.

This plurality obstructs the positive overclaim that the operational reconstruction selects an interpretation. The registered claim P5-C06 is instead a negative scope statement. It remains **OPEN**until the full project signature and prose have passed their final audit. The narrower syntactic observation, that the current Paper 5 signature contains no ontology or collapse field, is exact. Assumption P5-A13 remains **UNVALIDATED**because it is the governance condition for the complete artifact, not a physical postulate.

18 Interference in the executable witness

The Haskell suite includes one precisely defined two-path calculation. For complex amplitudes $a, b \in \mathbb{C}$, define

$$I(a,b) = |a+b|^2 - |a|^2 - |b|^2.$$

For $a = 1$ and $b = -1$,

$$I(1,-1) = -2.$$

The function rejects nonfinite real or imaginary components and rejects overflow in the squared magnitudes.

This calculation is **COMPUTATIONAL**. It assumes complex amplitudes. It is not a reconstruction of amplitudes from the operational bridge and is not evidence for `P5-C02`. Its purpose is to keep the meaning of "interference" explicit whenever the word occurs in executable evidence.

19 Counterexamples as acceptance tests

19.1 Labels admit many evaluations

Take a two-label set $\{0, 1\}$. It can be paired with the distribution $(1/2, 1/2)$, with $(3/4, 1/4)$, or with no probability map. The label constructors do not choose among these possibilities. The generic Haskell type `LabelEvaluation` pairs a nonempty list of phantom-typed outcome labels with a normalized distribution of the same length. One test attaches both distributions above to the identical typed label list and checks that the evaluations differ. The constructor validates only nonemptiness, dimension, finiteness, nonnegativity, and normalization. The weights remain supplied data. This witness is **COMPUTATIONAL**and is separate from the same-POVM, different-instrument witness of `CE-P5-10`. This is `CE-P5-06`.

19.2 Tomography and a qubit are not the full theorem

A theory may have a qubit-shaped object and some locally accessible statistics while lacking state completeness, homogeneity, self-duality, or factorizable composite forms. Barnum-Wilce needs the complete ambient class. This is `CE-P5-07`.

19.3 Complex direct sums

The algebra

$$M_2(\mathbb{C}) \oplus M_3(\mathbb{C})$$

is a finite-dimensional complex C-star algebra. Its center is nontrivial, so it carries superselection sectors. Complex selection alone does not reduce it to one matrix factor. This is `CE-P5-08`.

19.4 One POVM, many instruments

Theorem 7.1 gives infinitely many instruments for the same projective POVM by varying the prepared states σ_i. Outcome probabilities do not determine update. This is `CE-P5-10`.

19.5 Gleason begins after reconstruction

The source type of theorem 9.1 already contains a real or complex Hilbert space and its projection lattice. It cannot be used to derive either item from Paper 3. This is `CE-P5-11`.

19.6 Interpretive plurality

Everett, Bohm, and GRW add different semantic or dynamical structures to an operational quantum calculus. That calculus does not choose among them. This is `CE-P5-13`.

20 Formal and executable evidence

20.1 Lean scope

The Lean module `FoundationalReformulation/Quantum.lean` defines:

- a non-enforcing finite schema projection named `OperationalBridge`;
- caller-selected carrier types, raw object-level operations, and unconstrained proposition slots in that projection;
- finite rational distributions and bounded effects;
- finite row-stochastic channels;
- channel application, identity, and sequential composition;
- independent product distributions;
- deterministic coarse-graining;
- the twenty-three separate Paper 5 assumption fields;
- the exact **OBSTRUCTED**record for `P5-C05`.

The schema projection does not define a category, a symmetric monoidal category, a functor, coherence data, test-indexed transformations, or the full operational equivalence relations of theorem 3.1. Its proposition slots are not among the kernel-checked results below.

The kernel checks:

- nonnegativity and upper bounds of finite effect evaluation;
- normalization of channel application;

- normalization of channel composition;
- identity action;
- agreement of channel composition with sequential application;
- normalization of product distributions;
- identity coarse-graining.

No axiom, opaque definition, classical shortcut, noncomputable declaration, `sorry`, or `admit` appears in the project Lean sources.

20.2 Haskell scope

The module `src/Foundational/Quantum.hs` uses validated finite `Double` arrays. Its public constructors permit malformed adversarial values, while every operation revalidates inputs. The suite tests:

- the same typed probability-free label list with two distinct valid normalized evaluations;
- normalized distributions and row-stochastic channels;
- sequential identity and associativity;
- product normalization;
- coarse-graining and composition of coarse maps;
- bounded effect evaluation;
- same-probability, different-update instruments;
- the explicit two-path interference function;
- rejection of negative weights, empty carriers, dimension mismatch, invalid maps, NaN, infinities, and aggregate overflow;
- behavior on both sides of the exported numerical tolerance;
- acceptance of IEEE negative zero as the number zero.

QuickCheck explores finite examples. It does not prove the CDP principles, complete positivity at all ancillary dimensions, cone homogeneity, self-duality, Gleason, or scalar classification.

20.3 Evidence matrix

Statement	Status	Evidence or blocker
Finite effect evaluation lies in $[0, 1]$	**PROVED**	`evaluateEffect_nonnegative`; `evaluateEffect_atMostOne`.
Finite channel application preserves normalization	**PROVED**	`FiniteChannel.apply`.
Finite identity and sequential channel laws	**PROVED**	`FiniteChannel.identity_apply`; `FiniteChannel.comp_apply`.
Finite product distribution normalizes	**PROVED**	`tensorDistribution`.
Floating-point finite witness suite	**COMPUTATIONAL**	Haskell tests, including the distinct label-evaluation witness and malformed and nonfinite values.
`P5-C01`	**OPEN**	No operational bridge from a physical Paper 3 model.
`P5-C02`	**OPEN**	No validation of `P5-A01` through `P5-A09`.
`P5-C03`	**OPEN**	No HSD model, factorizable or dagger structure, complete composite family, or qubit construction.
`P5-C04`	**OPEN**	No project proof of the projection or all-effects noncontextuality hypotheses.
`P5-C05`	**OBSTRUCTED**	`CE-P5-12` and the incompatible case table.
`P5-C06`	**OPEN**	The negative scope statement awaits the complete signature and prose audit.

Table 3: Formal, computational, open, and obstructed results.

21 Why the assumptions remain unvalidated

The mathematical operational bridge has been defined on paper as a target interface, but no instance has been constructed from the proposed physical contexts. The Lean record with the same name is only a non-enforcing finite schema projection. Even an inhabitant would not supply the missing categories, functor, coherence, test transformations, equivalence relations, or operation-tied laws. Its proposition slots can be chosen independently of its operations and do not require proofs. The record therefore supplies no evidence for `P5-A01` through `P5-A03`.

The finite channel lemmas establish closure properties for a classical control fragment. They do not establish that the operational quotients of the intended theory are finite-dimensional ordered vector spaces.

The CDP principles are universal physical and operational conditions. Testing a finite collection of stochastic channels cannot establish them for every system and state.

The Barnum-Wilce route needs cone automorphism, inner-product, composite, and qubit data absent from Paper 3. The paper does not relabel a symmetry-invariant algebraic state as an HSD state cone.

The Gleason and all-effects premises require a Hilbert/effect event domain that is reconstructed only conditionally in the main route. Their noncontextuality assumptions have not been derived.

The dynamics and instrument assumptions are separate because a transformation class neither chooses time nor attaches a unique update to an effect.

22 Prior art and project delta

22.1 Operational reconstructions

Hardy's reconstruction is an important historical comparator [15]. Its final version uses five axioms, a parameter-counting result $K = N^r$, simplicity, and continuous reversible transformations. The continuity axiom excludes the classical branch. Hardy does not recover a particular Hamiltonian. Schack shows that the limiting-frequency axiom can be removed under a Bayesian reformulation [20].

CDP is selected as the main route because it produces the finite complex state, effect, transformation, instrument, and channel structure in one operational framework and includes qubits internally. The project contribution is not a new proof of CDP. It is the explicit typed boundary between Paper 3 and CDP, the separate axiom registry, and the preservation of all failure cases.

22.2 Jordan comparison

Barnum-Wilce provides a clean view of scalar selection and composite hypotheses. Barnum-Graydon-Wilce shows why relaxing local tomography reopens real and quaternionic possibilities. The project keeps these as one comparison branch rather than importing an HSD conclusion into the CDP proof.

22.3 Probability representation

Gleason, Busch, and CFMR answer a downstream representation question. They do not build the operational framework. The paper's delta is to prevent the dimension-two repair from being described as standard projection Gleason and to prevent Gleason from being used as a Hilbert reconstruction.

22.4 Compositional semantics

Categorical quantum mechanics explains protocol composition once the relevant categorical structure has been supplied. The operational bridge explains what extra data a reconstruction needs. The two uses of category theory are compatible but not interchangeable.

23 Dependency map

The main route is:

$$\begin{array}{c}\text{Paper 3 labels and representations}\\ \downarrow\ \mathsf{OperationalBridge}\\ \text{finite }\mathsf{OPT}\\ \downarrow\ \text{six CDP principles}\\ \text{finite complex quantum theory}\end{array}$$

Both arrows are dashed at the project level. The second is a proved literature implication once its source framework and principles are instantiated. The first is an unconstructed bridge.

The comparison route is:

$$\begin{array}{c}\text{finite state-complete HSD models}\\ \downarrow\\ \scriptstyle\text{factorization or dagger-HSD, composites,}\\ \scriptstyle\text{local tomography, and a qubit}\\ \downarrow\\ \text{complex C-star/Jordan systems with direct-sum sectors}\end{array}$$

No arrow derives its source from Paper 3.

The probability checks occur later:

$$\text{Hilbert/effect structure} \longrightarrow \begin{cases}\text{projection Gleason}, & \dim \geq 3,\\ \text{all-effects theorem}, & \dim = 2.\end{cases}$$

Time evolution and instruments branch independently:

$$\begin{aligned}\text{reversible channels} &\not\Longrightarrow \text{chosen one-parameter dynamics},\\ \text{POVM} &\not\Longrightarrow \text{chosen instrument}.\end{aligned}$$

24 Limitations

24.1 Finite main scope

The main theorem is finite-dimensional. The paper does not reconstruct infinite-dimensional Hilbert spaces, C-star algebras, W-star algebras, unbounded observables, normal states, sigma-weak continuity, or field-theoretic domains. The condensed structures of Paper 2 may eventually host some of this analysis, but they do not supply it automatically.

24.2 No constructed operational bridge

The source contexts, label functor, system category, and label-to-effect maps have not been jointly constructed. Operational equivalence and the finite ordered quotients have not been proved. This is the main blocker for `P5-C01`.

24.3 No validation of reconstruction principles

No project theorem establishes causality, perfect distinguishability, ideal compression, local distinguishability, pure conditioning, or purification for the intended contexts. They remain independent assumptions.

24.4 No scalar master theorem

Real and quaternionic theories are excluded only inside selected composite and tomography assumptions. Infinite-dimensional complex selection is not supplied. Direct-sum superselection remains in the Jordan comparison.

24.5 No selected dynamics

The reconstruction classifies allowed reversible channels but chooses no Hamiltonian. A spacetime realization from Paper 4 would still need a physical dynamics and covariance relation.

24.6 No collapse theorem

An instrument gives an operational conditional update. It does not decide whether that update describes a physical collapse. No interpretation is derived.

25 Conclusion

The route from representation data to quantum mechanics has a large, typed middle layer. Paper 3 outcome labels do not carry probabilities. An operational bridge must add systems, tests, transformations, states, effects, evaluation, equivalence, convexity, discarding, and finite ordered linear structure.

Once that bridge is available, the CDP theorem provides a strong finite-dimensional reconstruction under six distinct principles. The theorem yields complex density matrices, effects, CP trace-nonincreasing operations, channels, instruments, and unitary reversible maps. The project has not verified any of those bridge or principle assumptions, so claims `P5-C01` and `P5-C02` remain **OPEN**.

The Barnum-Wilce route is a separate comparison. It selects complex finite C-star/Jordan structure only inside its full state-complete HSD, factorization or dagger, composite, tomography, and qubit hypotheses. It keeps direct-sum superselection. Claim `P5-C03` remains **OPEN**.

Gleason applies after Hilbert structure and excludes qubits. The qubit trace form uses the stronger all-effects and POVM-noncontextual domain. Claim `P5-C04` remains **OPEN**.

No one audited theorem covers real, quaternionic, low-dimensional, infinite-dimensional, and superselection cases. Claim `P5-C05` is **OBSTRUCTED**. Reversibility does not choose time evolution, and a POVM does not choose an instrument. The operational calculus does not choose a measurement interpretation. Claim `P5-C06` remains **OPEN**pending the complete project audit.

The Lean proofs and Haskell tests establish a smaller finite algebraic core. That core is useful because it makes normalization, composition, coarse-graining, and counterexamples executable. It remains exactly where it belongs: below the imported reconstruction theorems, with no claim that computation has replaced their hypotheses.

A Complete assumption inventory

ID	Assumption	State
`P5-A01`	Explicit operational bridge signature, including labels-to-effects.	**UNVALIDATED**
`P5-A02`	Serial, parallel, equivalence, coarse-graining, convex, and discarding laws.	**UNVALIDATED**
`P5-A03`	Finite-dimensional ordered operational quotients and normalized closed-circuit evaluation.	**UNVALIDATED**
`P5-A04`	Causality and unique deterministic effects.	**UNVALIDATED**
`P5-A05`	Perfect distinguishability.	**UNVALIDATED**
`P5-A06`	Ideal compression.	**UNVALIDATED**
`P5-A07`	Local distinguishability.	**UNVALIDATED**
`P5-A08`	Pure conditioning.	**UNVALIDATED**
`P5-A09`	Purification with fixed-purifier essential uniqueness.	**UNVALIDATED**
`P5-A10`	Projection-measure Gleason hypotheses in dimension at least three.	**UNVALIDATED**
`P5-A11`	All-effects and finite-POVM noncontextuality for qubits.	**UNVALIDATED**
`P5-A12`	One theorem covers every scalar, dimensional, and superselection case.	**UNVALIDATED**
`P5-A13`	No ontology, beable, preferred outcome, observer cut, or collapse law.	**UNVALIDATED**
`P5-A14`	Finite state-complete order-unit models.	**UNVALIDATED**
`P5-A15`	Homogeneous state cones.	**UNVALIDATED**
`P5-A16`	Supplied self-dualizing inner products.	**UNVALIDATED**
`P5-A17`	Factorizable self-duality or complete dagger-HSD data.	**UNVALIDATED**
`P5-A18`	Allowed non-signalling locally tomographic composites for every relevant pair.	**UNVALIDATED**
`P5-A19`	An actual Jordan qubit $M_2(\mathbb{C})_{\mathrm{sa}}$.	**UNVALIDATED**
`P5-A20`	Exact Piron lattice and rank hypotheses.	**UNVALIDATED**
`P5-A21`	Exact Soler form and infinite-sequence hypotheses.	**UNVALIDATED**
`P5-A22`	Separately chosen strongly continuous dynamics.	**UNVALIDATED**

ID	Assumption	State
P5-A23	Separately supplied instrument or update process.	**UNVALIDATED**

Table 4: Every Paper 5 assumption remains unvalidated.

B Source locator record

CDP. The checked source is arXiv version 3 of [8]. The operational framework is in Section II. The five axioms and purification postulate are stated separately. The state-space conclusion is Theorem 20, and the transformation conclusion is Corollary 52.

Barnum-Wilce. The checked source is arXiv version 2 of [3]. The relevant results are Proposition 1 for the factorizably HSD route and Proposition 16 for the dagger-HSD categorical route. The conclusion includes superselection rules.

Gleason and qubits. The original projection-measure result is Theorems 3.5 and 4.1 of [13]. Busch's properties P1-P3 and theorem supply the all-effects route [6]. The finite-dimensional POVM frame theorem is Theorem 1 of [7].

Quantum logic. The modern exact statement of Piron's representation theorem is taken from [24]. The scalar restriction and the importance of the infinite orthonormal sequence are recorded from [17, 21].

Alternative composites. The universal tensor-product table and categorical examples are in [4]. They support the claim that non-locally-tomographic real and quaternionic systems remain coherent alternatives.

Processes and interpretation. Instrument provenance comes from [9, 18]. The interpretation boundary uses [5, 10, 12] only to show the plurality of additional structures.

C Audit checklist

1. Paper 3 labels remain probability free.
2. The mathematical operational bridge is explicit and unvalidated.
3. The Lean record is only a non-enforcing finite schema projection.
4. Its proposition slots are caller supplied, operation independent, and unverified.
5. Systems, tests, outcomes, transformations, states, effects, and evaluation have separate fields.

6. Serial and parallel composition have separate laws.
7. Operational equivalence is a congruence obligation.
8. Convexity and discarding are explicit.
9. Finite ordered state and effect spaces are explicit.
10. The six CDP principles have separate assumption IDs.
11. CDP is the only main reconstruction route.
12. The CDP theorem is imported, not Lean formalized.
13. General operations are trace nonincreasing; channels are trace preserving.
14. Effects, POVMs, operations, instruments, and channels are distinct.
15. Barnum-Wilce is a separate comparison.
16. Its state-complete, HSD, factorization or dagger, composite, tomography, and qubit hypotheses are all present.
17. Direct-sum superselection is retained.
18. Standard Gleason is downstream and has dimension at least three.
19. Qubits use all-effects and POVM noncontextuality.
20. Real, complex, quaternionic, dimension-two, infinite, and superselection cases have separate rows.
21. `P5-C05` is **OBSTRUCTED**everywhere.
22. Reversible maps do not become a selected dynamics.
23. POVMs do not become unique instruments.
24. Instrument update does not become an interpretation.
25. All twenty-three assumptions are **UNVALIDATED**.
26. Finite Lean and Haskell evidence has no reconstruction-theorem label.

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